(12) **EUROPEAN PATENT APPLICATION**

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(54) A trigonometric transform function generator.

(57) A single carrier is amplitude-modulated by a signal which is a transform (as the sine function) of the audio signal, and one set of sidebands is removed. In a receiver (76), the signal can be recovered by first deriving the Hilbert transform of the sine transform, and multiplying it by the signum of the derivative of the original audio signal to obtain the cosine transform. The sine and cosine transforms are then decoded to obtain the original audio signal.

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TRANSFORM MODULATION SYSTEM

Background of the Invention

This invention relates to the field of radio trans-
missions systems and, more particularly, to a system for
providing all of the advantages of present FM systems
within one-half the normal bandwidth.

5   It is well known that angular modulation systems
have advantages over amplitude modulation systems, not-
ably in the areas of signal-to-noise ratio and in the
ability to "capture" a receiver. However, due to the
steadily increasing demand for more communication chan-

10   nels throughout the radio spectrum, the smaller bandwidth
required by AM-SSB systems has begun to appear more
desirable in spite of the considerable disadvantages.
Thus, there has developed a need for a system having the
advantages of FM within the bandwidth of an AM-SSB

15   system.
Known angular modulation systems are of two main
types. One system uses direct frequency modulation of an
oscillator which could not be a stable oscillator such as
a crystal oscillator except with a very low modulation

20   index. The usual phase modulated system uses a stable
oscillator with narrowband modulation, then multiplies
the frequency of the modulated signal to obtain the de-
sired broadband signal.

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Summary of the Invention

It is therefore an object of the present invention
to provide a transmission system having the advantages
of FM transmission and approximately one-half the band-
width.

Another object is to provide an improved system of
FM transmission.

These objects and others which will become apparent
are obtained in a system wherein the modulating signal is
one of the transform signals of an FM system, with one
set of sidebands removed. The second transform signal
can be recovered at the receiver to provide the original
input signal.

Brief Description of the Drawing

FIG. 1 is a block diagram of an FM system.

FIGS. 2A, B and C are block diagrams of three
embodiments of the decoder of FIG. 1.

FIG. 3 is a block diagram of still another embodi-
ment of the decoder of FIG. 1.

FIG. 4 is a block diagram of the transform modula-
tion system with waveforms.

FIG. 5 is a more detailed diagram relating to
FIG. 4.

FIG. 6 is a chart of waveforms related to FIGS. 5
and 8.

FIG. 7 is a block diagram of a portion of the
diagram of FIG. 5.

FIG. 8 is a block diagram of an additional element
to be used with FIG. 7.

FIG. 9 is a comparison chart of system characteris-
tics.

FIG. 10 is a block diagram related to the trans-
mitter portion of FIG. 5.

Detailed Description of the Preferred Embodiments

Background Theory

In order to facilitate a better understanding of the specific embodiments of the present invention, it may be preferable to begin by discussing the existing systems of modulation as used with analog signals. These fall into
5   two basic categories, each with its own advantages and disadvantages. Amplitude modulation as used in standard broadcasting, is a double sideband signal (AM-DSB) system including the carrier frequency and requires a bandwidth double the audio bandwidth since there is a sideband on
10   each side of the carrier frequency for each frequency in the modulating signal. Suppressing the carrier frequency (AM-DSB-SC) improves the signal-to-noise ratio by as much as 6 dB. Single sideband AM transmission, as used for other than the broadcast band, reduces the required
15   bandwidth by half, is typically used with a suppressed carrier (AM-SSB-SC), and the signal-to-noise ratio is better than with AM-DSB or AM-DSB-SC. The second basic method of modulation is angular modulation which may be either frequency (FM) or phase (PM) modulation. In
20   angular modulation, only the frequency (or phase) of the carrier frequency is varied by the modulating signal. The required bandwidth depends on the modulating index and, for normal broadcasting, is at least ten times the audio bandwidth which provides a significant improvement
25   in signal-to-noise ratio. This may be termed wideband FM, usually referred to merely as FM. In narrowband FM (FM-NB) the modulating index may be as low as 1.0, making the bandwidth approximately the same as for AM-DSB, but the signal-to-noise ratio is still greater than for any
30   form of AM assuming that both AM and FM use pre-emphasis and de-emphasis.

In transform modulation (TM) the advantages of both AM and FM are combined; i.e., for any given transmission bandwidth, the signal-to-noise ratio will be considerably greater with TM than with any form of AM or FM (PM) and

5   the "capture" characteristic of FM is retained. This can best be seen by first recognizing that an FM signal can be represented by either:

(1) $a = A_c \sin [\omega_c t + m_f \sin \omega_a t]$

where A_c is the carrier amplitude, ω_a is $2\pi f_a$, the audio frequency, and m_f is $\Delta F/f_a$ where ΔF is the peak fre-

10   quency deviation, or by

(2) $a = A_c \{ [\cos (m_f \sin \omega_a t)] \sin \omega_c t +$

          $[\sin (m_f \sin \omega_a t)] \cos \omega_c t \}$

Equation 2 is thus seen to be a form of double sideband, suppressed carrier, quadrature modulated AM system with both sidebands required for coherent decoding of the original input signal. In distinction to known AM quad-

15   rature signals, however, each of the modulating signals, $\cos (m_f \sin \omega_a t)$ and $\sin (m_f \sin \omega_a t)$ is a "transform" of the original audio signal multiplied by the modulation index. Thus, each of the modulating signals has a limited peak magnitude, limited by the maximum values of sine

20   and cosine. This amounts to "folding" each signal back upon itself. It is this folding process which provides both the capture characteristic and the higher signal-to-noise ratio of FM since, when the signal is "unfolded" the level of any noise introduced during transmission

25   remains unchanged.

In order to provide an ideal signal it is necessary to determine what function of the original signal, when used to modulate the carrier single sideband, would allow coherent decoding at a receiver. Such a function must

30   have the following characteristics: It must somehow "fold" the signal back upon itself in order to maintain a constant peak amplitude of the carrier; it must do the "folding" in a manner which produces the minimum number

of sidebands (narrowest possible bandwidth); and, of
course, it must be a function which is possible to obtain
and to decode.

As will become apparent, the ideal way to obtain
such a function is to transform the signal $m_f \sin \omega_a t$
to $\sin (m_f \sin \omega_a t)$ which has a constant peak amplitude
for values of m_f greater than $\pi/2$. When a single carrier
is amplitude modulated with this transform signal (herein
termed "TM" for "transform modulation"), and one set of
sidebands is removed before transmission, the signal-to-
noise ratio of the received signal is the same as an FM
signal with the same modulation index (and twice the
bandwidth) and better than an AM signal of the same band-
width.

FIG. 1 illustrates how an understanding of the equa-
tion (2) as given above can be used to provide a new trans-
mission system for standard FM. A signal $f(t)$ is coupled
from a terminal 10 to two function generators 12, 14, which
provide the complementary functions $f_1(t)$ and $f_2(t)$ which
are $\sin f(t)$ and $\cos f(t)$ which will be referred to herein-
below. The signals from function generators 12, 14 may be
coupled through lowpass filters 15 which would pass all fre-
quencies necessary to allow recovery of $\sin f(t)$ and $\cos$
$f(t)$ with satisfactory accuracy. These signals are coupled
as inputs to a form of AM quadrature system 16 (shown in
dashed line) and are, within the limits of the transmis-
sion system, the output signals of the system 16. The sys-
tem 16 includes an encoding or modulation portion having a
frequency source 18 for providing $\sin \omega_c t$, a phase shifter
20 for producing a quadrature signal $\cos \omega_c t$, a multiplier
22 coupled to function generator 12 and frequency source
18, a multiplier 24 coupled to function generator 14 and
phase shifter 20, and an adder 26 coupled to combine the
outputs of multipliers 22, 24. The adder output is
connected by a link 28 to a demodulation portion of the
system 16 which may contain a frequency source 30, a

phase shifter 32, a multiplier 34 coupled to the link 28
and source 30, a multiplier 36 coupled to the link 28 and
the phase shifter 32. The frequency source 30 may be
synchronized with the frequency source 18. It will be
5 seen that the outputs of the multipliers 34, 36, are
essentially $f_1(t)$ and $f_2(t)$. The FM signal in the link
28 can be an RF signal which may be amplified, translated
in frequency and transmitted, thus the link 28 may be
merely one wire or a complete transmitting/receiving
10 system.

    Keeping in mind that the modulating signals were not
simply f(t), the input signal at terminal 10, but the
sine and cosine functions of f(t), it will be seen that
the signal at the link 28 is the signal of Equation 2
15 above, a constant amplitude, frequency-modulated signal.
A decoder 38 receives the two signals sin f(t) and
cos f(t) and derives f(t). It will, of course, be
understood that the link 28 could couple to any other
form of standard FM receiver.

20   FIGS. 2A, B and C are possible embodiments of the
decoder 38 of FIG. 1. In FIG. 2A, the $f_1(t)$ signal is
coupled to a differentiating circuit 40 with output
signal [(df(t)/dt)] cos f(t) coupled to a divider 42. In
the divider, this signal is divided by cos f(t) and the
25 output is coupled to an integrator 44 with output signal
f(t) which is the original input at terminal 10 of
FIG. 1.

    In FIG. 2B, the output of the differentiator 40 is
coupled to one input of an op amp 46, the output of which
30 is coupled to the integrator 44. The output of the op
    which functions as a comparator
amp 46 is also coupled back to a multiplier 48. The
second input of the multiplier 48 is the input signal
$f_2(t)$, and the feedback forces the output of the multi-

plier, coupled to the op amps, to equal the differentia-
tor output. Thus, the input and output signals of the
integrator 44 are as in FIG. 2A.

In FIG. 2C, $f_2(t)$ is coupled to the op amp 46, and
5   the amplifier output is coupled back to a multiplier 50.
The signal $f_1(t)$ is coupled to a second input of multi-
plier 50, and the output is coupled to another integrator
52. The integrator 52 output is thus forced to be $f_2(t)$
and is coupled to the op amp 46 and, again, the integrator
10   44 functions as above.

In FIG. 3, sin $f(t)$ + any extraneous noise is repre-
sented by "Q" and cos $f(t)$ + noise by "I" for simplicity
in the drawing figure. As is known, the frequency modu-
lation on the carrier is the rate of change of carrier
15   phase angle ϕ or

$$\frac{d\phi}{dt} = \frac{I \frac{dQ}{dt} - Q \frac{dI}{dt}}{Q^2 + I^2}$$

Deriving this rate-of-change signal usually requires a
limiter/discriminator combination. To implement the
derivation of the modulating signal $f(t)$, the signal Q is
squared in a multiplier 53 and signal I is squared in a
20   multiplier 55. Signals I^2 and Q^2 are summed in an adder
56 and the sum is coupled to a divider 58. Signal Q is
also coupled to a multiplier 59 and to a differentiator
60, the output of the latter being coupled to a multiplier
62. Signal I is also coupled to the multiplier 62 and to
25   a differentiator 64, the output of the latter being coupled
to the multiplier 59. The output signals of multipliers
59 and 62 are coupled to a subtractor 66 and the subtrac-
tor output signal is thus

$$I \frac{dQ}{dt} - Q \frac{dI}{dt}$$

It will be seen that the combination of multipliers 59,
30   62, differentiators 60, 64 and subtractor 66 ~~(within~~
~~dashed line 67)~~ performs the discriminator function. This

signal is coupled to the divider 58 having an output
of dϕ/dt and the integral of that signal, from the inte-
grator 44, is f(t), the original signal. It will be seen
that the decoder of FIG. 3 detects only frequency (or

5   phase) modulation and ignores amplitude variations in the
signal caused by noise or interference and has all the
characteristics of other FM systems.

FIG. 4 illustrates a system (TM) providing the S/N
ratio and "capture" characteristics of FM systems, in-

10   cluding that of FIG. 1, but requiring only one-half the
bandwidth of a comparable FM signal. The input signal at
terminal 68 is f_3(t) or m_f sin ω_at, shown as a sinusoidal
waveform 70, where m_f is the modulation index. The
output of the function generator 12 is now sin[f_3(t)] or

15   f_4(t). This latter output signal is shown in the wave-
form 72 and is "folded back" upon itself within the
constraints of the sine values $\pm$1. The waveform 72 is
the modulating signal coupled to a single sideband (SSB)
transmitter 74. The transmitted signal may be received

20   by an SSB receiver 76 with extraneous amplitude modula-
tions caused by noise or interference. Therefore, the
demodulated signal 78 may be f_4(t) + n, n = noise, rather
than being identical to waveform 72. When the waveform
78 is "unfolded" in a function generator 80 which

25   essentially derives the arc sine of the generator input
signal, the output at a terminal 82 is the waveform 84,
f_3(t) + n. Thus, as in angular modulation, the original
waveform 70 has been restored with only a slight amount
of noise, giving a signal-to-noise ratio improvement

30   factor of m_f, and with the important advantage that one
single set of sidebands produces the same improvement
over AM as does FM with the same modulation index and
twice the bandwidth.

The diagram of FIG. 5 shows many similarities to

35   FIG. 1; i.e., an input signal f(t) at the terminal 10 is
transformed in the function generator 12 to sin f(t),
filtered in filter 15, and coupled to the system 16, and

the sin f(t) output of the system 16 is coupled to the
decoder 38.   However, instead of coupling the signal f(t)
to the second function generator 14 (as in FIG. 1), the
sin f(t) output from function generator 12 is also
5   coupled to a phase-shifting network 88 which is not fre-
quency selective and which shifts the phase of each audio
component by 90°, an operation known mathematically as
taking the Hilbert transform of a signal. The output
signal of the network 88 is then coupled to the multi-
10   plier 24, making the composite signal at the link 28 an
SSB signal. This method of modulating a DSB-SC transmit-
ter with the modulating signal on one input terminal and
the Hilbert transform of the modulating signal on a
second terminal is known as the phase-shift method of
15   generating an SSB signal, thus the signal at link 28 is
an SSB transmission of sin f(t) or TM. When the TM sig-
nal is demodulated in a quadrature detector, one output
is sin f(t) and the other, from the multiplier 36 is the
Hilbert transform of sin f(t). The output signal of the
20   multiplier 36 is coupled to a phase-inverting switch 90,
which is controlled by a control signal sgn $[df(t)/dt]$
from a terminal 92, producing an output signal which is
approximately cos f(t). The control signal will be
discussed with respect to FIG. 7. As in FIG. 1, the
25   sin f(t) and cos f(t) signals are processed in the
decoder 38 to provide an output of f(t). In this circuit
it would be desirable, as is known in the art, to send a
small signal at the carrier frequency of source 18
through the link 28 for the purpose of synchronizing the
30   source 30 by the usual phase locked means. This small
amount of carrier can also be used for AGC as in AM-SSB.

In the waveforms of FIG. 6, line A represents
sin f(t) where $f(t) = m_f \sin \omega_a t$, $\omega_a t$ is a single tone
audio frequency and m_f is the modulating index (10
35   radians in this example). The "folding" effect of the
sine-of-the-sine function is apparent in line A. Line B
represents the Hilbert transform of the signal of Line A.

Line C is the signal at terminal 92 and may be expressed
as sgn[df(t)/dt], representing ±1, depending on the sign
of the given function. While line D is approximately cos
f(t) the more accurate cos f(t) signal of line E is to be
5   preferred, and a means of deriving line E will be shown in
and described with respect to FIG. 8.

In FIG. 7, a terminal 94 couples the signal sin f(t)
from the multiplier 34 to the decoder 38. A terminal 96
couples the Hilbert transform signal from multiplier 36
10   to the phase-inverting switch 90 and also to a zero
crossing detector 97 comprising a limiter 98, a differen-
tiator 100 and a full wave rectifier 102. The detector
97 detects the zero crossings of the signal at the termi-
nal 96, and is coupled to one input of an AND gate 104.
15   A terminal 106 of the decoder 38 provides a signal
df(t)/dt to a full wave rectifier 108. The rectified
output is coupled to a differentiator 110 and to an adder
112 which has a second input from the differentiator 110.
The combined output of the adder 112 thus includes small
20   positive pulses which precede the zero crossings of the
signal from the terminal 106. These positive pulses
trigger a one-shot multivibrator 114, the output of which
is coupled to a second input of the AND gate 104. The
output of the AND gate then represents only those zero
25   crossings in the output of the detector 97 which coincide
closely with the zero crossings of the signal at terminal
106. The coincident crossings are shown within the
windows 116 indicated in line B of FIG. 6. The AND gate
output triggers a flip-flop 118 which provides the signal
30   of line C of FIG. 6.

In FIG. 8, a coherence circuit 120 has been added between the
phase-inverting switch 90 and the decoder 38 (FIG. 7) to
prevent the output signal of the switch 90 from approach-
ing zero at the same time the signal at terminal 94 goes
35   to zero, since this event would cause the decoder 38 to
lose coherence and produce a large burst of noise. An

output terminal 122 of decoder 38 provides a signal
$(I^2 + Q^2)$ to a comparator 124 having a reference terminal
126. The comparator output is coupled to a phase-
inverting switch 128 which is controlled by a signal
(sgn [cos f(t)]) from a F/F 130. The signal from the
phase-inverting switch 90 is combined with the output of
the phase-inverting switch 128 in an adder 132, the out-
put of which is essentially cos f(t) as seen in FIG. 6E.
The adder output is coupled to decoder 38 and, through a
limiter 134 and a differentiator 136, to the F/F 130, pro-
viding a signal sgn [cos f(t)] which controls the switch
128. When the signals at terminals 94 and 96 both approach
zero, the comparator 124, through the switch 128, pro-
vides a signal which brings the cos f(t) signal to the
proper polarity. In this circuit the small carrier signal
referred to with respect to FIG. 5 can also be rectified
and amplified to provide the reference voltage at terminal
126 of the comparator 124. With this type of reference
voltage, the receiver can determine when there is no
transmission, and squelch as necessary.

FIG. 9 is a chart comparing the characteristics of
AM-SSB, FM and TM and is based on the foregoing explana-
tion and drawing.

The block diagram of Fig. 10 relates to the trans-
mitter portion of FIG. 5 and is included here for further
understanding of the transmitter operation. The function
generator 12 of FIGS. 1 and 5 has an input signal f(t)
and an output signal sin f(t). It may be seen that, with
an FM signal at the link 28 of FIG. 1, the output of the
multiplier 34 is also sin f(t). In FIG. 10, the block
140 represents an Armstrong type of frequency modulation
generator, which produces a signal such as the signal
at link 28 of FIG. 1, and includes a carrier frequency
source 142, a phase shifter 144, a multiplier 146, an
adder 148 and a frequency multiplier 150. The frequency
source 30 derives a synchronous reference signal from the

carrier source 142 and the frequency multiplier 152 is
essentially identical to the frequency multiplier 150 of
the Armstrong type frequency modulation generator 140.
The output signal of the multiplier 150 is coupled through
5   a phase shifter 154 to the multiplier 34 to provide the
output signal sin f(t). The block 140 is therefore the
equivalent of blocks 12, 15, 18, 20, 22, 24, 26 and 88
(FIG. 5). It is apparent that, lacking the phase shifter
154, the output of the multiplier 34 would be cos f(t),
10   as is the output of the multiplier 36 or the function
input to multiplier 24 of FIG. 1.

     Thus, there has been shown and described a new
system of modulation which can provide all of the desir-
able characteristics of both AM and FM with minimum band-
15   width requirements, and a new way of providing standard
FM has been disclosed. Other modifications and varia-
tions are possible, and it is intended to cover all such
as fall within the spirit and scope of the appended claims.

     What is claimed is:

CLAIMS

1. A sine function generator comprising:

an FM signal generator coupled to receive an input information signal; and

demodulator means for deriving the sine of the information signal from the FM signal.

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Fig. 1

$f_1(t) = \text{SIN } f(t)$

DIFFERENTIATOR *40*

38

$f_2(t) = \text{COS } f(t)$

÷ *42*

INTEGRATOR *44*

$f(t)$

Fig.2A

$f_1(t)$

DIFFERENTIATOR *40*

46

INTEGRATOR *44*

$f(t)$

$f_2(t)$

X *48*

38

Fig.2B

$f_2(t)$

46

INTEGRATOR *44*

$f(t)$

$f_1(t)$

X *50*

INTEGRATOR *52*

38

Fig.2C

Fig. 3

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$f_3(t) = m_f \sin w_a t$

$f_4(t) = \sin(m_f \sin w_a t)$

$f_4(t) + n = \sin(m_f \sin w_a t) + NOISE$
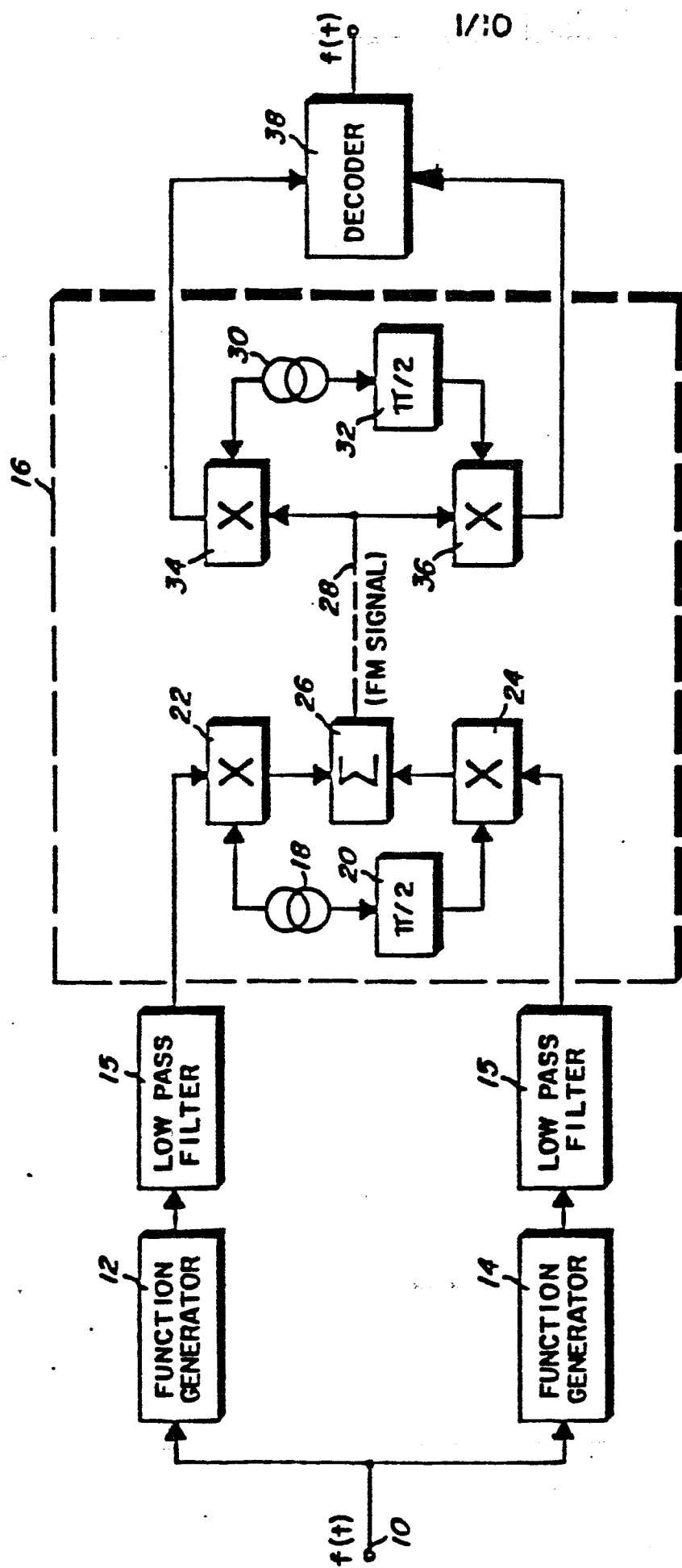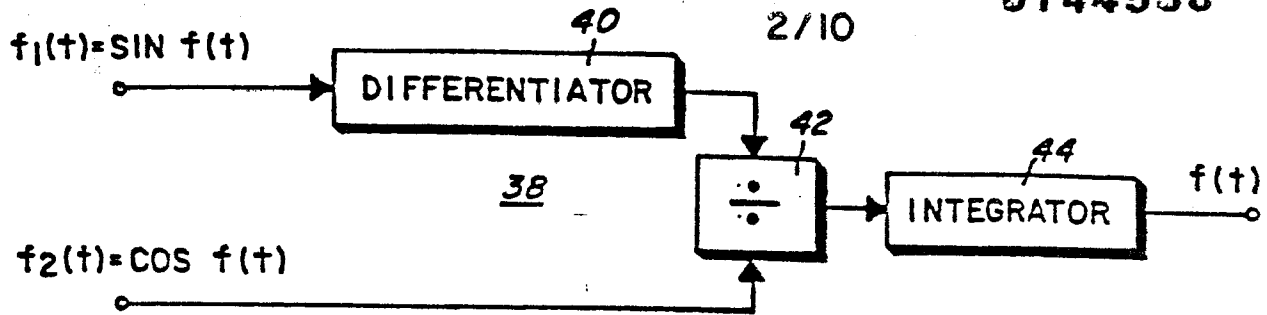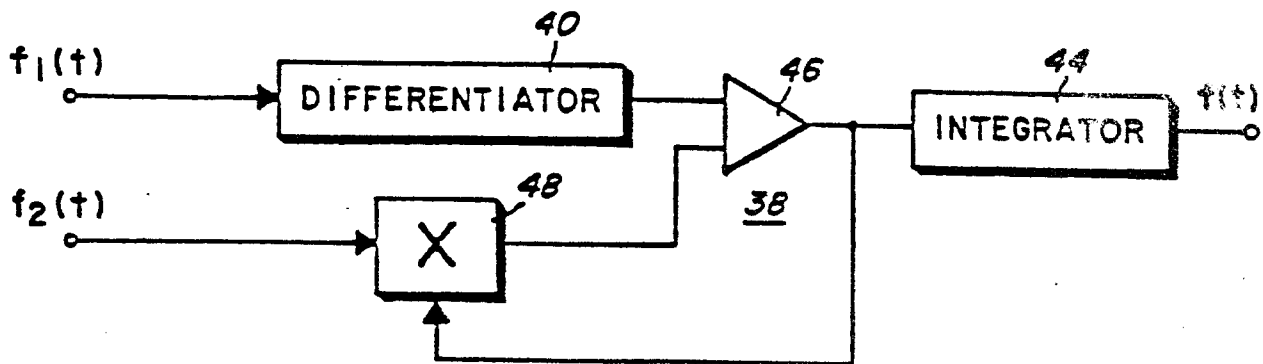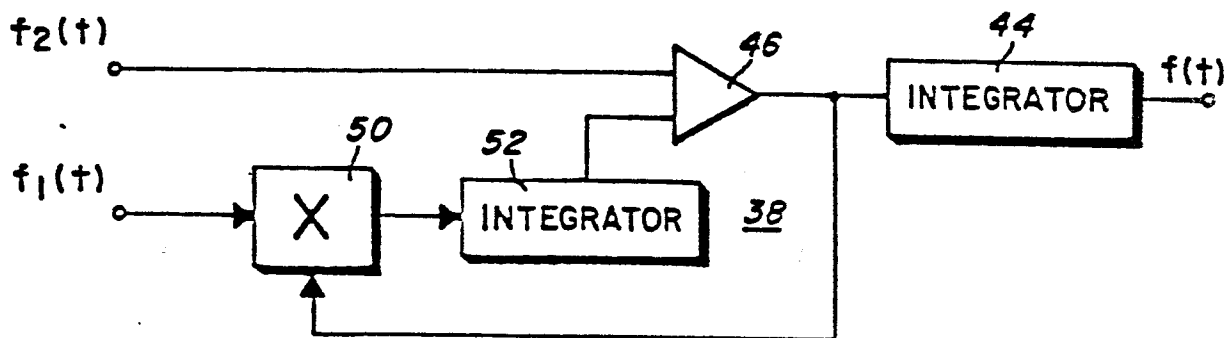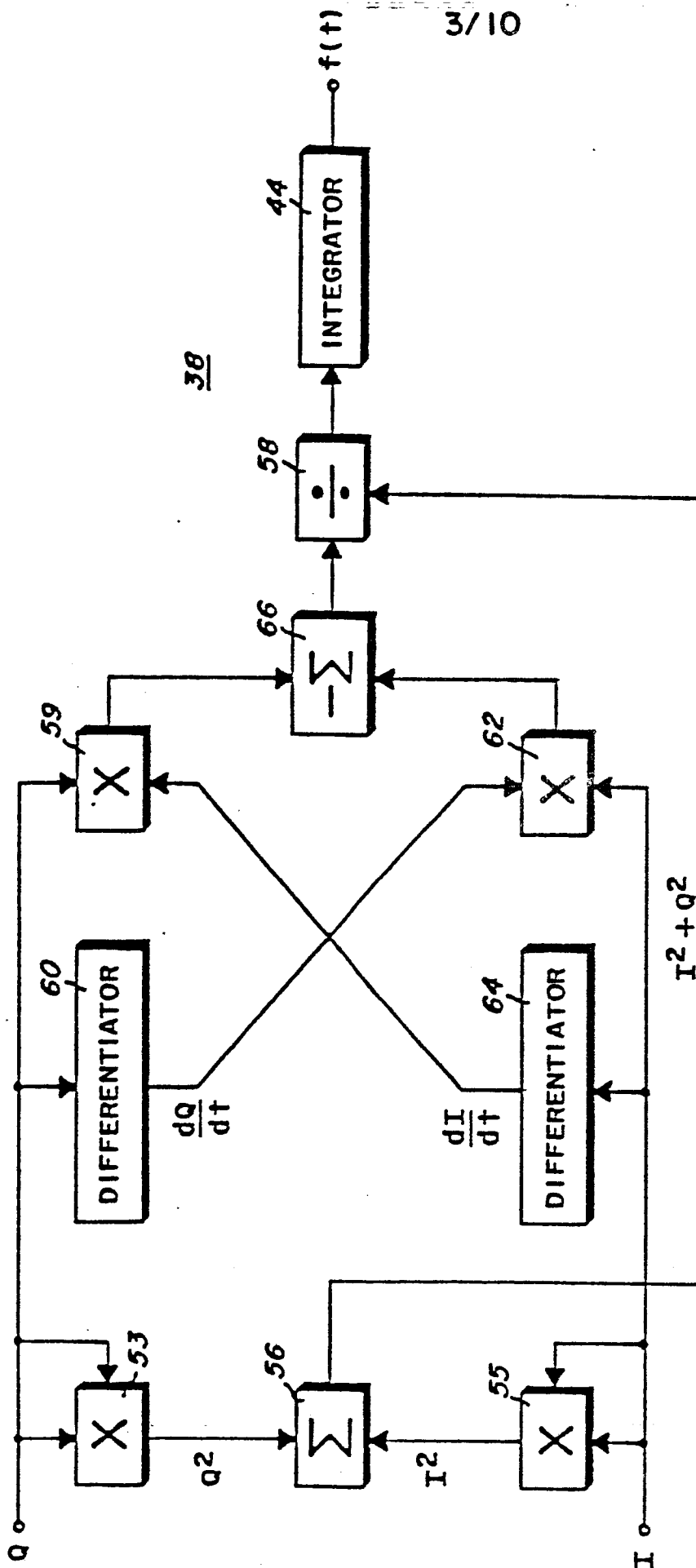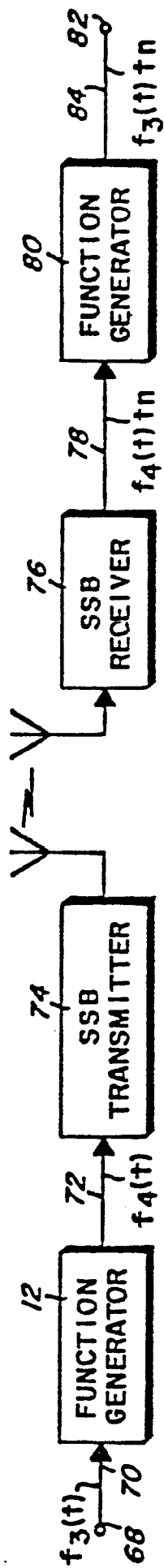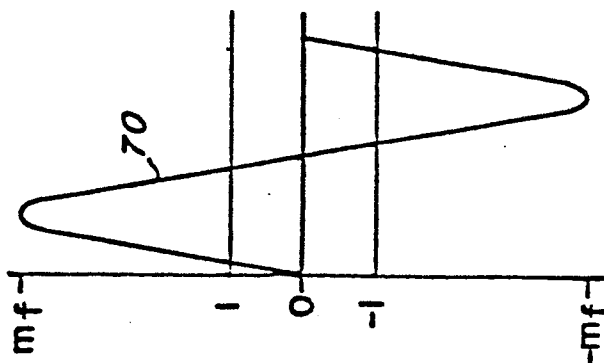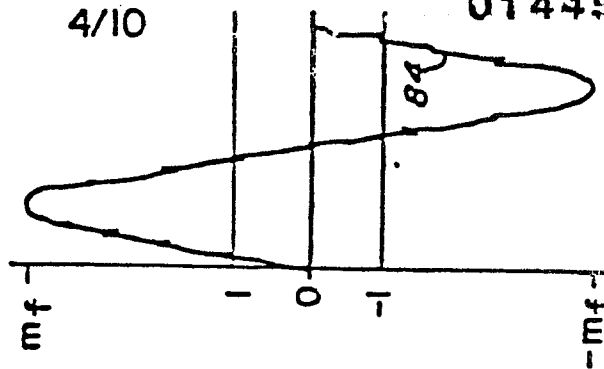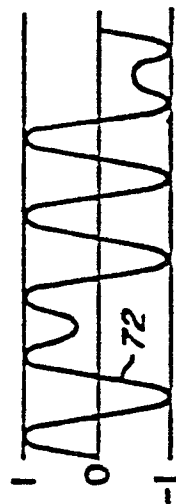
$f_3(t) + n = m_f \sin w_a t + NOISE$

$S/N \text{ OF } f_3(t)+n = S/N \text{ OF } \left[f_4(t)+n \right] m_f$

Fig. 4

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FIG. 5

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E/10

+1
0
-1

FIG.6C

2
1
0
-1
-2

0°   90°   180°   270°   360°

FIG.6D

2
1
0
-1
-2

0°   90°   180°   270°   360°

FIG.6E

1.5
1
.5
0
-.5
-1
-1.5

0°   90°   180°   270°   360°

FIG.6A

2
1
0
-1
-2

116

116

0°   90°   180°   270°   360°

FIG.6B

FIG. 7

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FIG. 8
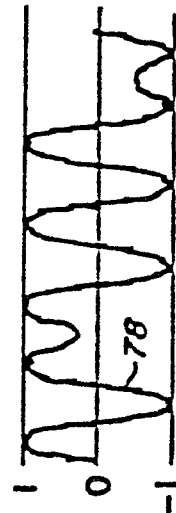
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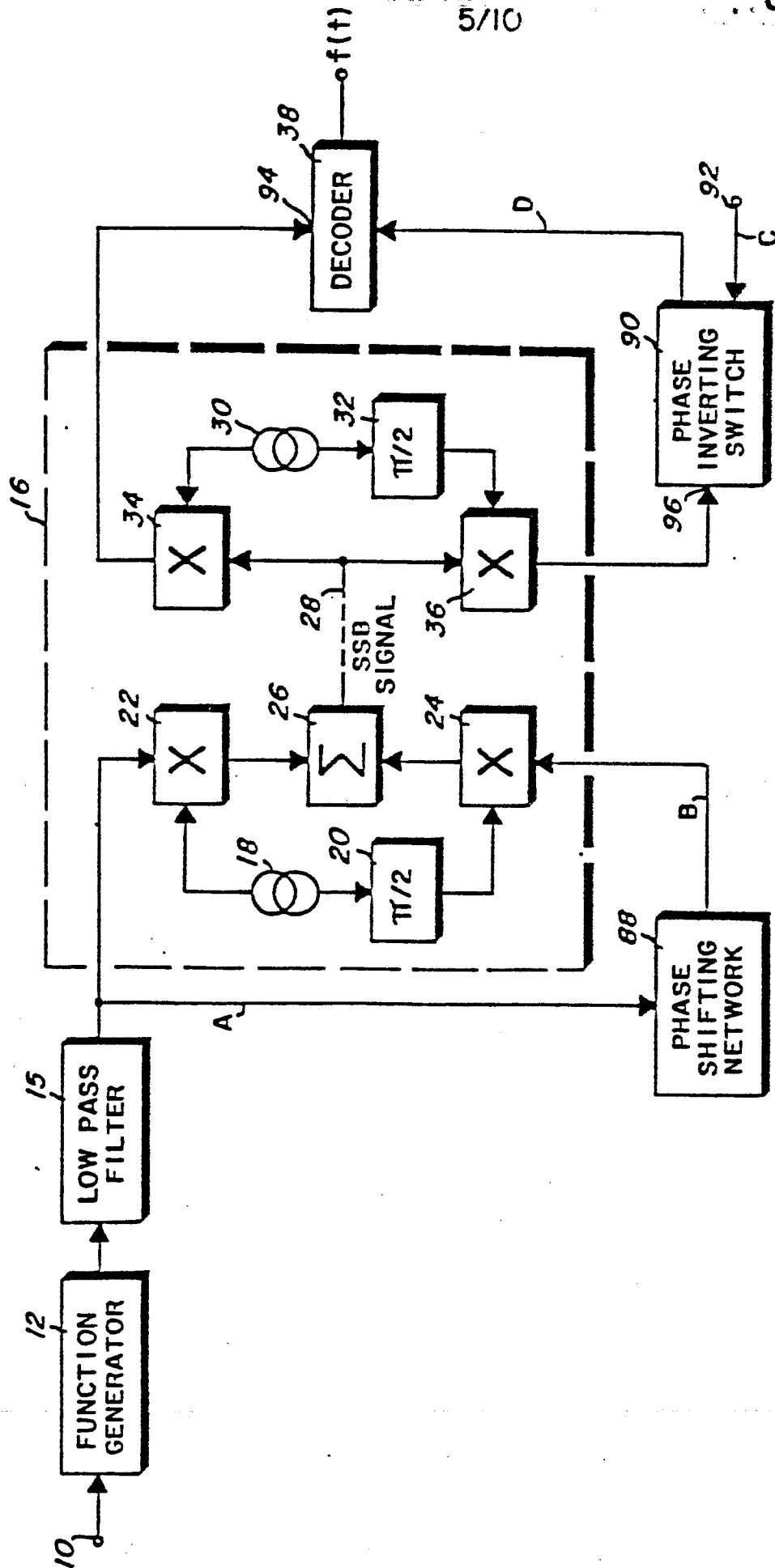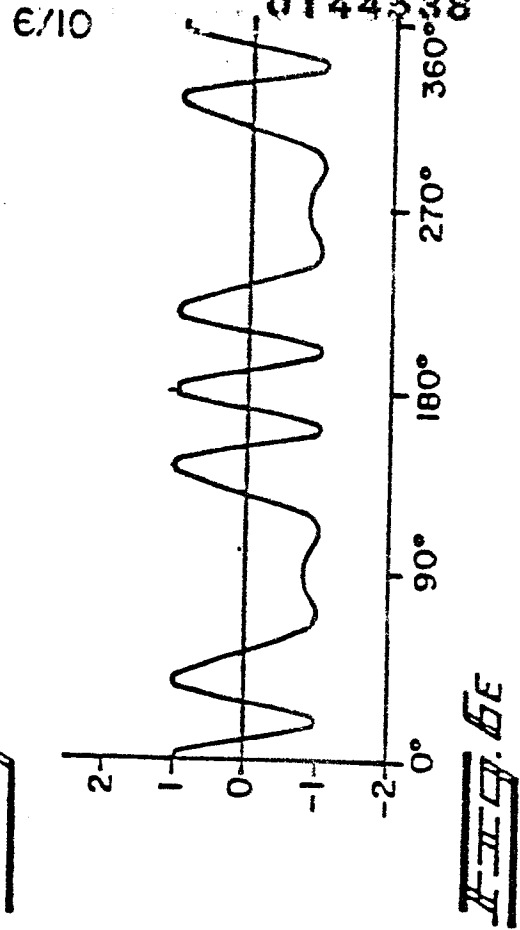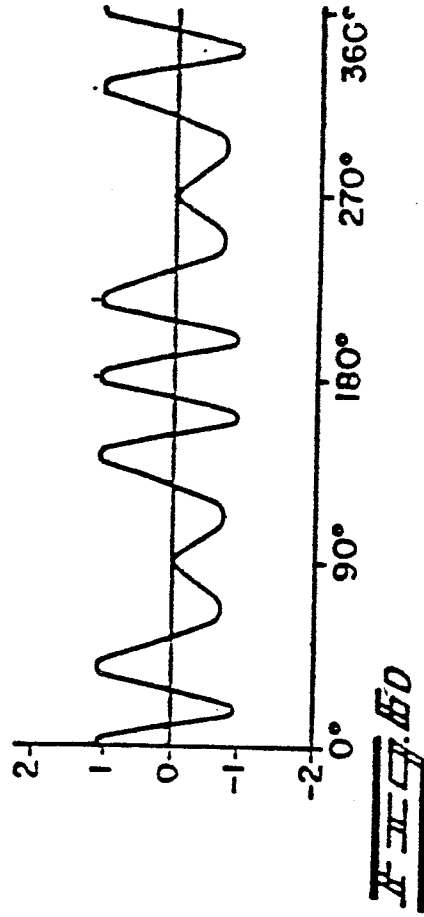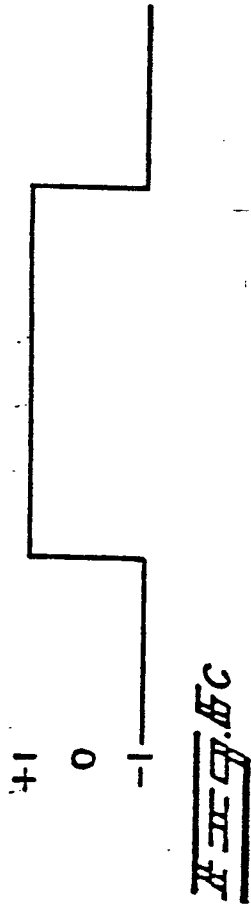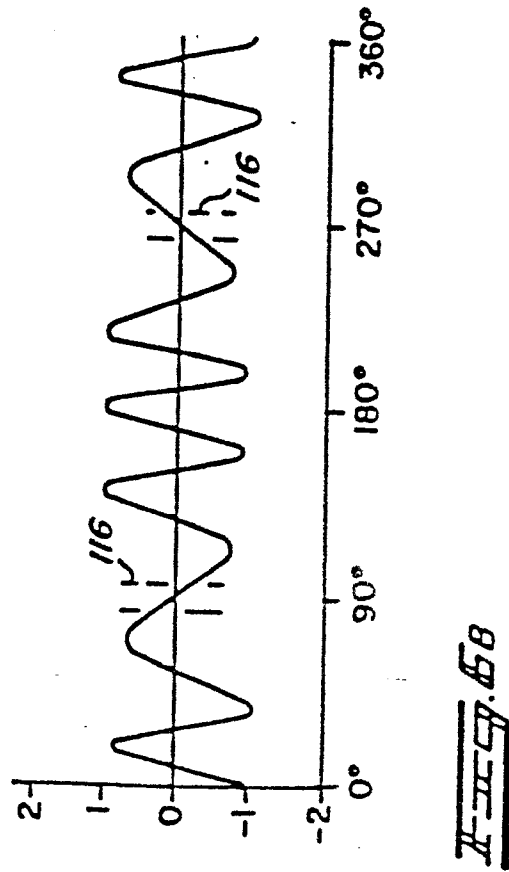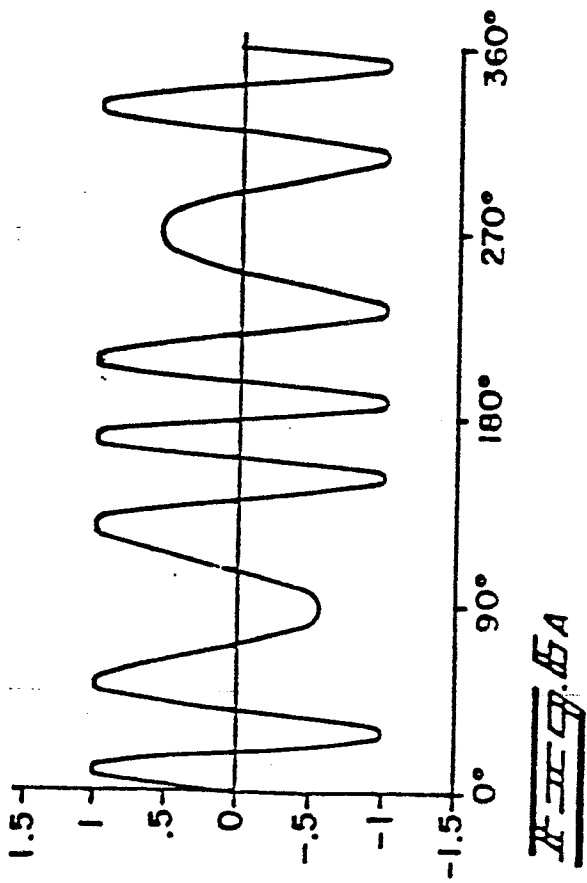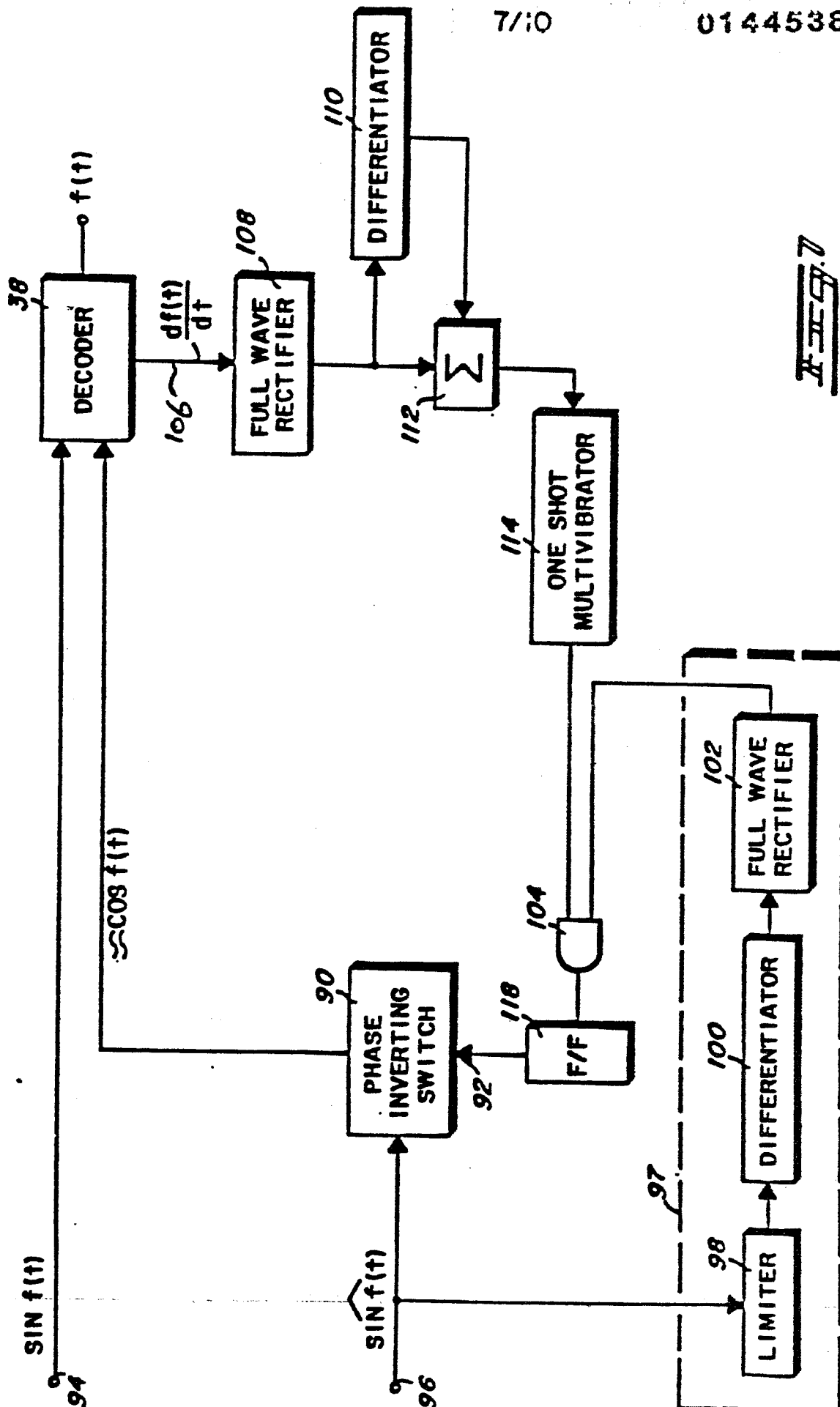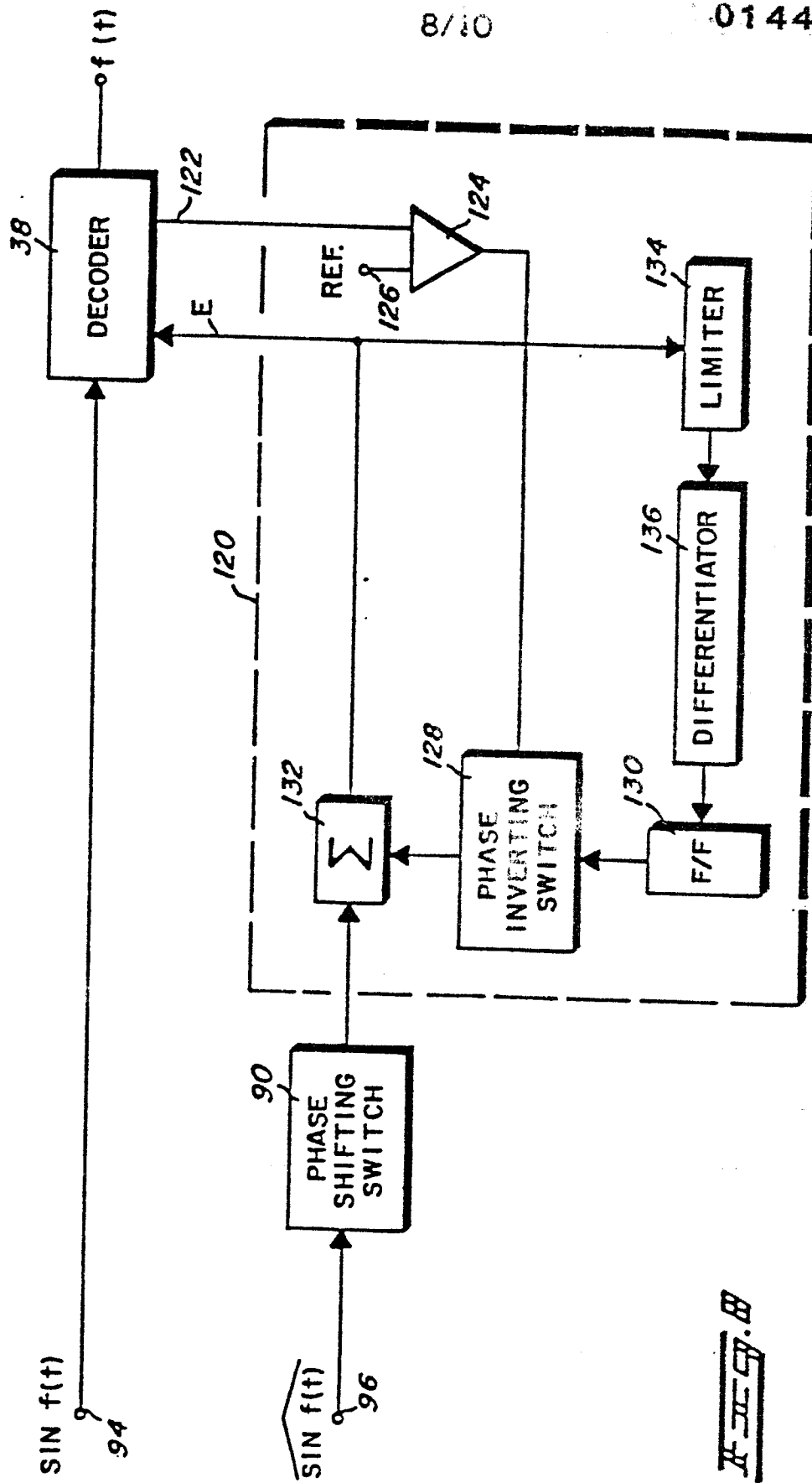
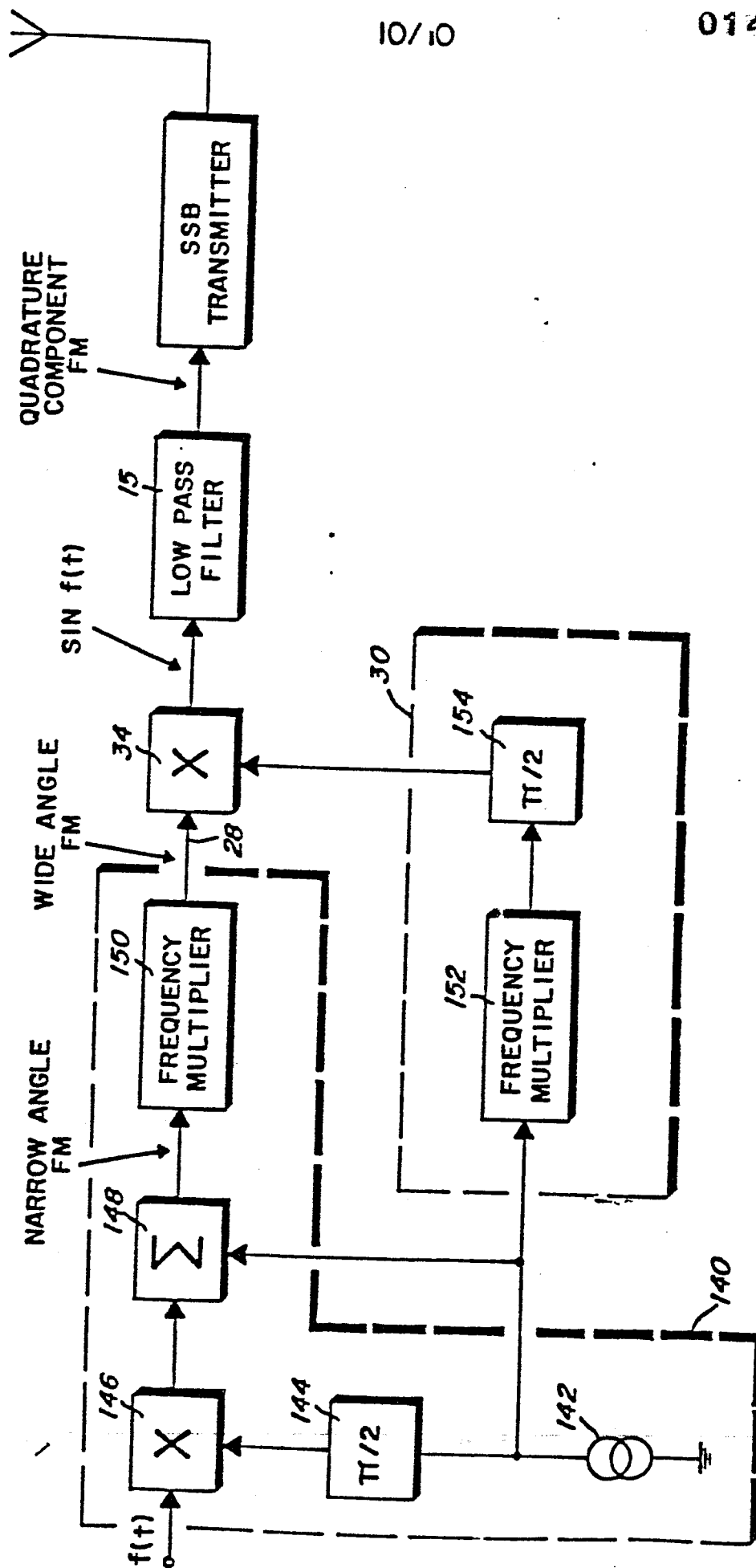|		SSB-AM MODULATION	FM MODULATION	TRANSFORM MODULATION
1	CAN SYSTEM BE DESIGNED SO THAT RECEIVER IS "CAPTURED" BY STRONGEST SIGNAL?	NO	YES	YES
2	IS AUDIO OUTPUT S/N > CARRIER S/N?	NO	YES	YES
3	IS AUDIO NOISE SPECTRUM TRIANGULAR $(N \propto f)$ OPTIMUM FOR USE WITH PRE-EMPHASIS/DE-EMPHASIS?	NO	YES	YES
4	CAN THE SYSTEM BE DESIGNED SO THAT THE R.F. SPECTRUM BANDWIDTH = AUDIO SPECTRUM BANDWIDTH?	YES	NO	YES
5	IS THE CARRIER SURPRESSED WHEN THERE IS NO MODULATION PRESENT?	YES	NO	YES
6	IS THE AMPLITUDE OF THE RADIATED SIGNAL CONSTANT?	NO	YES	APPROX. CONSTANT WHEN MODULATED
7	NEGLECTING TRANSMITTED IM—CAN THE BANDWIDTH OF THE R.F. SIGNAL BE LIMITED BY AUDIO FILTERING?	YES	NOT DIRECTLY	YES
8	CAN THE SYSTEM BE DESIGNED TO EXCHANGE S/N RATIO FOR BANDWIDTH?	NO	YES	YES

FIG. 9

Fig.10