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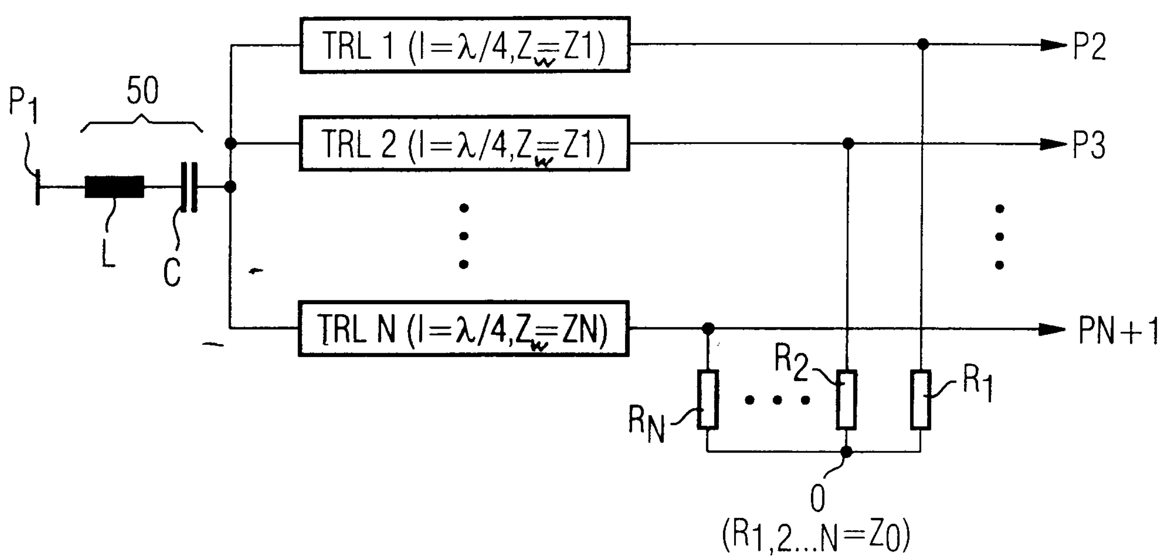
# (54) Wilkinson power divider circuit and corresponding design method

(57) The present invention provides a a Wilkinson power divider circuit, comprising a plurality of N transmission lines (TRL1, TRL2, ..., TRLN), N being an integer equal to or greater than 2, having a respective length of 1 = $\lambda_0/4$ at a center frequency $f_0$ , where $\lambda_0$ is the wavelength at $f_0$, and respective line impedances $Z_w$; said plurality of N transmission lines (TRL1, TRL2, ..., TRLN) being connected to a first port (P1) at a respective first end, to a respective second port (P2, P3, ..., PN+1) at a respective second end, and via a respective resistor ($R_1$, ..., $R_N$) to a node (O) at the respective second end. By means of an additional LC circuit (50) comprising at least an inductor having an inductance L and a capacitor having a capacitance C connected in series between said first port (P1) and said first ends of said plurality of N transmission lines (TRL1, TRL2, ..., TRLN) having its resonance frequency $f_r$ at or near said center frequency fo, it is possible to broaden the bandwith at a desired value of the minimum isolation.

FIG 1

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## Description

BACKGROUND OF THE INVENTION

**[0001]** The present invention relates to a Wilkinson power divider circuit and a corresponding Design Method, and more particularly to a Wilkinson power divider circuit, comprising a plurality of N transmission lines, N being an integer equal to or greater than 2, having a respective length of 1 = $\lambda_0/4$ at a center frequency $f_0$, where $\lambda_0$ is the wavelength at $f_0$, and respective line impedances $Z_w$; said plurality of N transmission lines being connected to a first port at a respective first end, to a respective second port at a respective second end, and via a respective resistor to a node at the respective second end.

**[0002]** In general, the conventional Wilkinson power divider circuit, which is usually fabricated in stripline or microstrip form, is a lossy multiport network that can be made having all ports matched with isolation between the output ports, although in a limited frequency range.

**[0003]** Wilkinson power divider circuits may be used as RF (radio frequency) power splitters or combiners, whose main feature is that they in theory provide perfect match to the reference impedance, as well as perfect isolation between the input or output ports - yet only in a limited frequency range. For more details, see for example chapter 8 in "Microwave Engineering" by David M. Pozar, Addison-Wesley, 1993, p. 395 ff.

**[0004]** The principle of the classical N-way equal Wilkinson power divider, as described in E. Wilkinson, "An N-way Hybrid Power Divider", IRE Trans. on Microwave Theory and Techniques. Vol. MTT-8, pp. 116 - 118, January 1960, is shown in Fig. 12, wherein Pl, P2, ..., PN+1 denote ports; TRL1, TRL2, ..., TRLN transmission lines; 1 = $\lambda_0/4$ respective line lengths at the center frequency $f_0$; $Z_w = \sqrt{N} x Z_0$ respective line impedances, $Z_0$ being the reference impedance (usually 50 ohms); O a node and $R_1$ ..., $R_N = Z_0$ resistors.

**[0005]** For power splitting purposes, port P1 is the input and ports P2, ..., PN+1 are the outputs, whereas for power combining purposes, port Pl is the output and ports P2, ..., PN+1 are the inputs. All ports P1, P2, ..., PN+1 are referenced to ground.

**[0006]** The term equal Wilkinson power divider means that in a power splitting application, power into port P1 is equally split to ports P2, ..., PN+1 and vice versa for a power combiner. It is also possible to make Wilkinsons with unequal power division/combining, see Pozar, section 8.3, pp. 399 - 400. A disadvantage of doing so is that outputs are matched to different impedances than $Z_0$.

**[0007]** The key parameters for RF(radio frequency) power splitters/ combiners are transmission loss ($|S_{21}|$, $|S_{31}|$, ...), reflection loss ($|S_{11}|$, $|S_{22}|$, ...), and especially isolation ($|S_{23}|$, ...). Of these, the reflection loss ($|S_{11}|$, $|S_{22}|$, ...) and the isolation ($|S_{23}|$, ...) are the most frequency dependent parameters.

**[0008]** In general, $S_{ij}$ is the S-parameter stating the ratio (in terms of amplitude and phase) to port i from an incoming electromagnetic wave at port j.

**[0009]** $S_{ij}$ is generally complex, and may thus be written as $Re\{S_{ij}\} + j \cdot Im\{S_{ij}\}$ or $|S_{ij}| \angle S_{ij}$. Here $Re\{S_{ij}\}$ is the real part of $S_{ij}$, $Im\{S_{ij}\}$ is the imaginary part, $|S_{ij}|$ is the magnitude, and $\angle S_{ij}$ is the angle. Thus, the following relations hold:

$$|S_{ij}| = \sqrt{(Re^2\{S_{ij}\}+Im^2\{S_{ij}\})}$$

$$\angle S_{ij} = \arctan(Im\{S_{ij}\}/Re\{S_{ij}\})$$

**[0010]** For example, $S_{11}$ is the reflection on port 1 (the contribution from port 1 to port 1). The corresponding return loss (in dB) is calculated from this value as $- 10 \cdot \log(|S_{11}|^2)$.

**[0011]** Similarly, the transmission gain of a 2-port device is $10 \cdot \log(|S_{21}|^2)$ (and of course the transmission loss is $-10 \cdot \log(|S_{21}|^2)$)..

**[0012]** In the case of the Wilkinson divider/combiner, an important parameter is the isolation, which for a 2-port Wilkinson is $-10 \cdot \log ((|S_{23}|^2)$ and $-10 \cdot \log(|S_{32}|^2)$ (for a symmetrical Wilkinson, these two expressions are identical).

**[0013]** The isolation is a measure of how much energy is leaked into port 2 when port 3 receives a certain amount of power-or vice versa.

**[0014]** For further informations on S-parameters, reference is made to section 5.4 of the above cited book by Pozar.

**[0015]** For example, a typical plot of the S-parameters of the classical 2-way (N = 2) equal Wilkinson power divider having one input and two output ports is shown in Fig. 13. The S-parameter curves were calculated using a simple computer design program for the analysis of microwave circuits.

**[0016]** Example values of the parameters are $f_0$ = 500 MHz, $Z_w = \sqrt{2} \times Z_0$ = 70.7 ohms, $Z_0$ being the reference impedance of 50 ohms, and R1 + R2 = 2 x $Z_0$ = 100 ohms.

**[0017]** As observed from Fig. 13, the reflection loss versus frequency behave similarly to the isolation, whereas the transmission loss is largely frequency independent. The useful isolation bandwidth $f_2$-$f_1$ at a minimum isolation of f.e. -30 dB of a Wilkinson is quite limited. This poses a problem in some applications, where broadband operation is required. It is possible to increase the bandwidth by using stepped multiple sections, but this requires more space and increases cost (see Pozar, sections 8.3, p. 400 - 401).

**[0018]** Thus, the technical problem to be solved is to provide an improved Wilkinson power divider circuit having an increased useful isolation bandwidth which may be easily constructed as well as a method of designing such improved Wilkinson power divider circuits having an extended bandwidth.

SUMMARY OF THE INVENTION

**[0019]** The present invention provides a Wilkinson power divider circuit as defined in claim 1 and a corresponding design method as defined in claim 6.

**[0020]** Particular advantages of the Wilkinson power divider circuit according to the invention are the increased isolation bandwidth and the inherent DC-decoupling at the port P1.

**[0021]** The principal idea underlying the present invention is that the isolation is very sensitive to the match on port 1, i.e. the reflection loss $|S_{11}|$, and not nearly as sensitive to the match on other ports (2, 3, etc.). Therefore, if a match with wider bandwidth is achieved on port P1, the isolation will also have a wider bandwidth. A simple series LC-circuit (coil L + capacitor C connected in series) having its resonance frequency $f_r$ at or near the center frequency $f_0$ of the isolation band is appropriate. It is in general appropriate to adjust or detune the characteristic impedance of the transmission lines and/or the LC-values, in order to achieve a symmetric response around said center frequency $f_0$.

**[0022]** Preferred embodiments of the present invention are listed in the respective dependent claims.

BRIEF DESCRIPTION OF THE DRAWINGS

**[0023]** The present invention will become more fully understood by the following detailed description of preferred embodiments thereof in conjunction with the accompanying drawings, in which:

Fig. 1 shows the principle of an N-way Wilkinson power divider circuit according to an embodiment of the present invention (for equal power split, Z1 = Z2 = ... = ZN);

Fig. 2 illustrates a typical plot of the S-parameters of the 2-way equal Wilkinson power divider according to another embodiment of the present invention having one input and two output ports in comparison to the classical 2-way equal Wilkinson power divider shown in Fig. 13;

Fig. 3 illustrates a typical plot of the S-parameters of a 2-way equal Wilkinson power divider having one input and two output ports before addition of the LC-circuit and optimization;

Fig. 4 illustrates a typical plot of the S-parameters of a 2-way equal Wilkinson power divider having one input and two output ports after addition of the LC-circuit and optimization;

Fig. 5 shows simulated results for isolation ($|S_{23}|_{max}$) versus a termination-resistance R11 for the ideal 2-way 50 ohm equal Wilkinson power divider shown in Fig. 3;

Fig. 6 shows a Smith chart (cfr. Pozar, section 3.4) from 200 to 800 MHz, for 3 values of $Z_w$: 60.7 ohms, 70.7 ohms ($\sqrt{2}$ x 50 ohms is optimum for the ordinary Wilkinson) and 80.7 ohms displaying how the input impedance of the Wilkinson behaves as a function of frequency for different transmission line characteristic impedances $Z_w$;

Fig. 7 shows a plot of $2\Delta f/f_0$ versus k;

Fig. 8 shows a plot of $\alpha$ versus k;

Fig. 9 isolation versus frequency for numerical example of another embodiment using equation (33) for C;

Fig. 10 isolation versus frequency for numerical example of another embodiment using equation (34) for C;

FIG. 11 isolation versus frequency for numerical example of the classical 2-way equal Wilkinson power divider having one input and two output ports;

Fig. 12 shows the principle of the classical N-way equal Wilkinson power divider; and

Fig. 13 illustrates a typical plot of the S-parameters of the classical 2-way equal Wilkinson power divider having one input and two output ports.

DESCRIPTION OF THE PREFERRED EMBODIMENTS

**[0024]** In the figures, identical reference signs denote identical or equivalent elements.

**[0025]** Fig. 1 shows the principle of an N-way Wilkinson power divider circuit according to an embodiment of the present invention.

**[0026]** According to Fig. 1, an LC-circuit 50 including an inductor L and a capacitor C connected in series between said first port Pl and said first ends of said plurality of N transmission lines TRL1, TRL2, ..., TRLN has been added to the conventional circuit shown in Fig. 12. Z1 ..., ZN denote line impedances which in a first approach equal the reference impedance $Z_0$ (of f.e. 50 ohms) times the square root of N.

**[0027]** Fig. 2 illustrates a typical plot of the S-parameters of the 2-way equal Wilkinson power divider according to another embodiment of the present invention having one input and two output ports in comparison to the classical 2-way equal Wilkinson power divider shown in Fig. 13.

**[0028]** The result of the addition of the tuned LC-circuit 50 regarding the isolation $|S_{23}'|$ is a considerably broadened isolation bandwidth $f_2' - f_1'$ at a minimum isolation of f.e. -30 dB, the maximum isolation at the center frequency $f_0$ being somewhat decreased. Moreover, a slightly enhanced refection loss on port P1 is observed.

**[0029]** However, in the most general case, the addition of the LC-circuit 50 has the effect that the response around said center frequency $f_0$ becomes asymmetric. Thus, it is necessary to adjust or detune the characteristic impedance of the transmission lines and the LC-values, in order to achieve a sufficiently symmetric response around said center frequency $f_0$ as well as a desired minimum isolation over the whole frequency band.

**[0030]** The latter procedure is an optimization procedure which can be performed on a standard computer design program for the analysis of microwave circuits. As a result, the resonance frequency $fr = 1/(2\pi\sqrt{LC}))$ of the LC-circuit 50 can slightly deviate from the center frequency $f_0$, and the impedance of the transmission lines from the reference value $\sqrt{N} \times Z_0$.

**[0031]** Fig. 3 illustrates a typical plot of the simulated S-parameters of a 2-way equal Wilkinson power divider having one input and two output ports before addition of the LC-circuit and optimization.

**[0032]** The example is based on a simple 2-way equal Wilkinson power divider. The center frequency $f_0$ is intended to be 0.5 GHz. Figure 3 shows the simulation result. Note that the two transmission lines have an electrical length of 90° (i.e. physical length equal to $\lambda/4$) at 0.5 GHz, with the characteristic impedance $Z_w$ being 70,7 ohms (= $\sqrt{2} \times 50$ ohms). The resistor between the right second ends of the two transmission lines is 100 ohms.

**[0033]** As becomes readily apparent, the simulated isolation bandwidth at a minimum isolation of -30 dB equals about 50 MHz.

**[0034]** Fig. 4 illustrates a typical plot of the simulated S-parameters of a 2-way equal Wilkinson power divider having one input and two output ports after addition of the LC-circuit and optimization.

**[0035]** First, with 0.5 GHz, one can calculate LC from the formula for series resonance frequency $fr = 1/(2\pi\sqrt{LC}))$. Choosing C = 10 pF, one obtains L ≈ IO nH. The result after manual optimization which is shown in Fig. 4 involves the following parameters:

$$Z_w = 73 \text{ ohms}$$

$$L = 15 \text{ nH}$$

$$C = 6.8 \text{ pF}$$

which is slightly different from the corresponding parameter starting values.

**[0036]** As becomes readily apparent, the simulated isolation bandwidth at a minimum isolation of -30 dB has dramatically increased to about 170 MHz, i.e. more than a factor of three.

**[0037]** Note that the peak isolation is lower than in the standard configuration, and that the isolation decreases more rapidly at very low and very high frequencies.

**[0038]** So far, the optimization of the parameter values has been performed in an empirical manner using a standard simulation program. In the following, analytical expressions will be derived to calculate the parameters L, C of the LC-circuit 50 and the impedances Z of the transmission lines.

**[0039]** Heretofore, the ideal 2-way 50 ohm equal Wilkinson power divider shown in Fig. 3 has been simulated.

**[0040]** Particularly, it has been investigated how the peak isolation changes as a function of source-impedance. Simulated results for $|S_{23}|_{max}$ at $f_0$ = 500 MHz versus a termination-resistance R11 is shown in Fig. 5, namely for R11 values ranging from 40 to 60 ohms. Notice the steep decrease in isolation when R11 deviates from the optimum 50 ohms.

**[0041]** From the plot, it is evident that the match on port 1 should be modified, if one wants to achieve isolation over a broader bandwidth. The match on port 2 and 3 is not as important for the isolation.

**[0042]** Next, it was investigated how the input impedance of the Wilkinson behaves as a function of frequency for different transmission line characteristic impedances $Z_w$.

**[0043]** The result is shown below in Fig. 6 in a Smith chart (cfr. Pozar, chapter 3.4) from 200 to 800 MHz, for 3 values of $Z_w$: 60.7 ohms, 70.7 ohms (optimum) and 80.7 ohms. One should notice that $S_{11}$ moves clockwise with increasing frequency, as for all other passive circuits, and should observe how the $S_{11}$-circle of the Wilkinson power divider expands and moves to the right as $Z_w$ increases.

**[0044]** It is known (f.e. see G. Gonzalez, ,,Microwave Transistor Amplifiers - Analysis and Design", section 6.4, p. 61, Prentice-Hall, 1984) that the input impedance $Z_{11}$ (or reflection $S_{11}$) of a series combination of $Z_0$ ohms resistive termination (the reference impedance), a coil L and a capacitor C will move entirely on the circle r = 1 with frequency, or in other words on the circle given by the normalized impedance z = 1 + jx, where $z = Z/Z_0$ and where x goes from $-\infty$ to $+\infty$ (cfr. to Fig. 6).

**[0045]** By choosing $Z_w > \sqrt{2} \times Z_0$ ohms for the Wilkinson power divider, it is therefore possible to obtain a conjugate match at two frequencies symmetrically located below and above $f_0$ = 500 MHz using the LC-series circuit 50, since the input impedances of the Wilkinson power divider at these two frequencies will be complex conjugates of each other (as detailed below). In general, conjugate matching is the optimum matching method (see for example section 3.6, p. 99 in Pozar).

**[0046]** From Fig. 6, it is expected that $S_{11}$ is symmetrical (but complex conjugated) around $f_0$. This is proved as follows.

**[0047]** The input impedance at port P1 of the Wilkinson power divider is equal to a parallel-connection of two identical transmission lines with impedance $Z_w$ and physical length $\lambda/4$ at $f_0$, since the resistor R has no effect when both ports P2 and P3 are terminated in $Z_0$.

**[0048]** The resulting impedance of two identical impedances Z in parallel is Z/2. Therefore, from transmission line theory, one obtains (f.e. see Pozar, page 79)

$$(1) \qquad Z_{11} = ½\cdot Z_w(Z_0+jZ_w\tan\beta 1)/(Z_w+jZ_0\tan\beta 1)$$

where $\beta = 2\pi/\lambda$ and 1 is the length of the transmission line. At $1 = \lambda_0/4$, as in this case, $\beta 1 = \pi/2$. Therefore, $Z_{11}$ at some positive amount $\Delta$ from $f_0$ may be rewritten as

$$(2) \qquad Z_{11}(\Delta) = ½\cdot Z_w(Z_0+jZ_w\tan(\pi/2+\Delta))/(Z_w+jZ_0\tan(\pi/2+\Delta))$$

and at some negative amount $-\Delta$ from $f_0$ as

$$(3) \qquad Z_{11}(-\Delta) = ½\cdot Z_w(Z_0+jZ_w\tan(\pi/2-\Delta))/( Z_w+ jZ_0\tan(\pi/2-\Delta))$$

**[0049]** Since $\tan(\pi/2-x) = -\tan(\pi/2+x)$, equation (5) may be rewritten as

$$(4) \qquad Z_{11}(-\Delta) = ½\cdot Z_w(Z_0-jZ_w\tan(\pi/2+\Delta))/Z_w-jZ_0\tan(\pi/2+\Delta))$$

**[0050]** In general, for any complex numbers $z_1$ and $z_2$ and their complex conjugates $z_1^*$ and $z_2^*$, the relation $(z_1/z_2)^* = (z_1^*/z_2^*)$ holds, so since $Z_w$ and $Z_0$ are real,

$$(5) \qquad Z_{11}(-\Delta) = ½\cdot Z_w[(Z_0+jZ_w\tan(\pi/2+\Delta))/ (Z_w+jZ_0\tan(\pi/2+\Delta))]^*$$

and therefore comparing (1) and (5), one arrives at

(6) $Z_{11}(-\Delta) = Z_{11}^*(\Delta)$

**[0051]** This means that if $Z_{11}$ = a+jb at the frequency $f_1 = f_0 - \Delta f$, then $Z_{11}$ = a-jb at the frequency $f_2 = f_0 + \Delta f$. As $S_{11}$ is closely related to $Z_{11}$ by $S_{11} = (Z_{11}-Z_0)/(Z_{11}+Z_0)$ then

$$S_{11}(-\Delta) = (Z_{11}(-\Delta)-Z_0)/(Z_{11}(-\Delta)+Z_0)$$
$$= (Z_{11}^*(\Delta)-Z_0)/(Z_{11}^*(\Delta)+Z_0) = S_{11}^*(\Delta)$$

**[0052]** So the same holds for $S_{11}$, i.e. $S_{11}$ is also symmetrical (but complex conjugated) around $f_0$.

**[0053]** The knowledge of the coordinates of the two points z = 1 +/- jα where the $S_{11}$-circle of the Wilikinson intersects with the r = f circle, makes it possible to calculate the necessary L and C, since their combined input impedance must be the complex conjugate of the Wilkinson splitter:

**[0054]** At $\omega_1$, $\omega_2$ we must therefore require that $Z_{LC} + Z_0 = Z_{11}^*$, where ZLC is the impedance of the LC circuit and $Z_0$ is the respective generator or consumer impedance, or equivalently:

(7) At $\omega_1 = 2\pi f_1$: $1/(j\omega_1 C) + j\omega_1 L + Z_0 = Z_0(1-j\alpha)$

(8) At $\omega_2 = 2\pi f_2$: $1/(j\omega_2 C) + j\omega_2 L + Z_0 = Z_0(1+j\alpha)$

**[0055]** It is necessary that the sign of jα is negative at $f_1 = f_0 - \Delta f$ and positive at $f_2 = f_0 + \Delta f$ in order to achieve the conjugate match.

**[0056]** As mentioned, the $S_{11}$-circle expands as $Z_w$ increases. Therefore, the potential bandwidth improvement also increases with $Z_w$.

**[0057]** Now, from (7), we will try to find Δf as a function of $Z_w$, by finding where $\mathrm{Re}\{Z_{11}\}/Z_0 = 1$:

(9) $Z_{11} = ½\cdot Z_w(Z_0+jZ_w\tan\theta)\ (Z_w-jZ_0\tan\theta))\ /\ [(\ Z_w+jZ_0\tan\theta)(Z_w-jZ_0\tan\theta)]$

where θ = β1, so

(10) $Z_{11} = ½\cdot Z_w/(Z_w^2+Z_0^2\tan^2\theta)\cdot[Z_0\cdot Z_w+Z_0\cdot Z_w\cdot\tan^2\theta+ j(Z_w^2\cdot\tan\theta-Z_0^2\cdot\tan\theta)]$

and therefore

(11)
$$\mathrm{Re}\{Z_{11}\} = {}^1/_2\cdot Z_w(Z_0Z_w+Z_0Z_w\tan^2\theta)/(Z_w^2+Z_0^2\tan^2\theta)$$
$$= {}^1/_2\cdot Z_0Z_w^2(1+\tan^2\theta)/(Z_w^2+Z_0^2\tan^2\theta)$$
$$= {}^1/_2\cdot Z_0(1+\tan^2\theta)/(1+(Z_o/Z_w)^2\tan^2\theta)$$
$$= {}^1/_2\cdot Z_0(1+\tan^2\theta)/(1+k^2\tan^2\theta)$$

defining $k = Z_0/Z_w$, where $0 < k < \sqrt{½}$, as we are only interested in $Z_0/\sqrt{2} < Z_w < \infty$ (f.e. see this document, page 13, last paragraph).

**[0058]** Finally,

(12) $\mathrm{Re}\{Z_{11}\}/Z_0 = ½\cdot(1+\tan^2\theta)/(1+k^2\tan^2\theta)$

**[0059]** Setting (12) equal to 1 in order to find 2Af means

$$(13)\qquad ½\cdot(1+\tan^2\theta)/(1+k^2\tan^2\theta) = 1 \Leftrightarrow 1+(2k^2-1)\tan^2\theta = 0$$

and therefore for $0 < k < \sqrt{½}$

$$(14)\qquad \tan^2\theta = 1/(1-2k^2) \Rightarrow \tan\theta = \pm 1\sqrt{}/(1-2k^2)$$

so

$$(15)\qquad \theta = \arctan(\pm 1/\sqrt{}(1-2k^2)) = \pm\arctan(1/\sqrt{}(1-2k^2)) = \pm\mathrm{Arctan}(1/\sqrt{}(1-2k^2)) + n\pi$$

where n is any integer number, and Arctan(x) denotes the principal value of arcus tangent of x, i.e. $|\mathrm{Arctan}(x)| \leq 2\pi$ for all x.

**[0060]** With $1 = \lambda_0/4 = c/(4f_0\sqrt{}\varepsilon_r)$ at $f_0$ where $\varepsilon_r$ is the relative dielectric constant of the transmission line medium, in general we have that

$$(16)\qquad 1/\lambda = (\lambda_0/4)/\lambda = c/(f_0\sqrt{}\varepsilon_r)/4(c/(f\sqrt{}\varepsilon_r)) = f/4f_0$$

and therefore

$$(17)\qquad \theta = \beta 1 = 2\pi 1/\lambda = \pi f/2f_0$$

**[0061]** Now, for any solution θ to (15), $\theta = \pm\xi + n\pi$, where $\pi/4 \leq \xi < \pi/2$ (i.e. $1/4 \leq \xi/\pi < ½$,

$$(18)\qquad \pi f/2f_0 = \pm\xi + n\pi \Leftrightarrow f = 2f_0(\pm\xi/\pi+n)$$

and since n is an integer, $1/4 \leq \xi/\pi < ½$, and $0 < f < 2f_0$, $2\Delta f$ is given by

$$(19)\qquad 2\Delta f = 2f_0([-\xi/\pi+1]-[\xi/\pi+0]) = 2f_0(1-2\xi/\pi)$$

and so

$$(20)\qquad 2\Delta f = 2f_0(1-2\mathrm{Arctan}(1/\sqrt{}(1-2k^2))/\pi)$$

i.e. $\Delta f = f_0(1-2\mathrm{Arctan}(1/\sqrt{}(1-2k^2))/\pi)$. Roughly put, $2\Delta f$ equals the isolation bandwidth.

**[0062]** A plot of $2\Delta f/f_0$ versus k is shown in Fig. 7.

**[0063]** In order to find α, we simply calculate $\mathrm{Im}\{Z_{11}\}/Z_0$ at $f = f_0 \pm \Delta f$:

$$(21)\qquad \mathrm{Im}\{Z_{11}\} = {}^1/_2\cdot Z_w(Z_w^2\tan\theta - Z_0^2\tan\theta)/(Z_w^2+Z_0^2\tan^2\theta) = {}^1/_2\cdot Z_w\tan\theta(1-k^2)/(1+k^2\tan^2\theta)$$

and since $k^2\tan^2\theta = k^2/(1-2k^2)$ at f = f0 ± Δf (from (14)) and $k = Z_0/Z_w$ we have

$$(22) \qquad \mathrm{Im}\{Z_{11}\} = {}^1/_2 \cdot Z_w \tan\theta (1-k^2)/(1+k^2/(1-2k^2))$$
$$= {}^1/_2 \cdot Z_w \tan\theta (1-2k^2)$$
$$= {}^1/_2 \cdot Z_w/\tan\theta$$
$$= \pm{}^1/_2 \cdot Z_w \sqrt{(1-2k^2)}$$
$$= \pm{}^1/_2 \cdot Z_0 \sqrt{(1/k^2-2)}$$

**[0064]** Therefore

$$(23) \qquad \alpha = {}^1/_2 \cdot \sqrt{(1/k^2-2)}$$
$$= {}^1/_2 \cdot \sqrt{((Z_w/Z_0)^2-2)}$$

**[0065]** A plot of $\alpha$ versus k is shown in Fig. 8.

**[0066]** In the following, it is derived how L and C are calculated.

**[0067]** From (1) and (2), one obtains

$$(24) \qquad \omega_1 L - 1/(\omega_1 C) = -X$$

$$(25) \qquad \omega_2 L - 1/(\omega_2 C) = X$$

where $X = \alpha Z_0$.

**[0068]** Defining $\Delta\omega = \omega_0 - \omega_1 = \omega_2 - \omega_0$ and $\varepsilon = \Delta\omega/\omega_0 = \Delta f/f_0$ means that

$$\omega_1 = \omega_0 - \Delta\omega = \omega_0 (1 - \varepsilon)$$

$$\omega_2 = \omega_0 + \Delta\omega = \omega_0 (1 + \varepsilon)$$

and therefore

$$(26) \qquad \omega_0(1-\varepsilon)L - 1/(\omega_0(1-\varepsilon)C) = -X$$

$$(27) \qquad \omega_0(1+\varepsilon)L - 1/(\omega_0(1+\varepsilon)C) = X$$

**[0069]** For $\Delta\omega << \omega_0$, i.e. $\varepsilon << 1$, (26) and (27) may be approximated by

$$(28) \qquad \omega_0(1-\varepsilon)L - (1+\varepsilon)/(\omega_0 C) = -X$$

$$(29) \qquad \omega_0(1+\varepsilon)L - (1-\varepsilon)/(\omega_0 C) = X$$

using the approximation $1/(1+\varepsilon) \approx 1-\varepsilon$ for $|\varepsilon| << 1$.

**[0070]** Adding (28) and (29) yields

$$(30)\qquad \omega_0 L - 1/(\omega_0 C) = 0 \Leftrightarrow \omega_0 L = 1/(\omega_0 C)$$

i.e. there is series resonance of L and C at $\omega_0 = 2\pi f_0$.

**[0071]** Similarily, subtracting (29) from (28) yields

$$(31)\qquad \omega_0 L\varepsilon + \varepsilon/(\omega_0 C) = X \Leftrightarrow \omega_0 L + 1/(\omega_0 C) = X/\varepsilon$$

**[0072]** By inserting (30) into (31), one obtains

$$(32)\qquad L = X/2\varepsilon\omega_0 = \alpha Z_0/2\varepsilon\omega_0$$

$$(33)\qquad C = 2\varepsilon/\omega_0 X = 2\varepsilon/\omega_0 \alpha Z_0$$

**[0073]** For large values of $\varepsilon$ (close to 1), the above two expressions cannot be used. Instead, using a similar approach, but without using the approximation, it can be shown that the expression for L remains unchanged, and that the expression for C becomes

$$(34)\qquad C = 2\varepsilon/(1-\varepsilon^2)\omega_0 X = 2\varepsilon/(1-\varepsilon^2)\omega_0 \alpha Z_0$$

**[0074]** So, with expression (32) and (34), the series connection of L and C is not necessarily in resonance at $\omega_0$. In fact, we obtain that the resonance frequency $\omega_r$ of L and C is given by

$$(35)\qquad \omega_r = 1/\sqrt{((1-\varepsilon^2)LC)} = \omega_0/\sqrt{(1-\varepsilon^2)}$$

so as $\varepsilon$ increases, $\omega_r$ also increases, away from $\omega_0$.

**[0075]** As a numerical example, with $f_0$ = 500 Mhz, $Z_0$ = 50 Ω and $Z_w$ = 80.7 Ω (> 70.7 Ω = √2 · 50 Ω), k becomes 0.6196, and $\Delta f$ becomes 285.9/2 = 142.95 MHz(eq. (20)), i.e. $f_1$ = 357.05 MHz and $f_2$ = 642.95 MHz. Furthermore, $\alpha$ becomes 0.3889 (eq.(23)), so that

$$\varepsilon = \Delta\omega/\omega_0 = \Delta f/f_0 = \tfrac{1}{2}\cdot 285.9/500 = 0.2859$$

$$X = \alpha\cdot 50\ \Omega = 0.3889\cdot 50\ \Omega = 19.445\ \Omega$$

$$\omega_0 = 2\pi\cdot 500\ \text{MHz}$$

**[0076]** Inserting into (32) and (33) yields L = 10.82 nH and C = 9.36 pF. Using expression (34) instead yields C = 10.19 pF, and $\omega_r = 2\pi\cdot 521.87$ MHz.

**[0077]** The resulting simulated isolation versus frequency for this embodiment is shown in Fig. 9 and 10, respectively, and for comparison the classical case in Fig. 11.

**[0078]** Assuming the same bandwidth for the case without LC circuit and with LC circuit having the above derived parameters, it can be simulated that the improvement in isolation using the approximate formulas (33) or (34) for C equals about 6.1 dB or 6.6 db, respectively, over the frequency range 357.05 to 642.95 MHz.

**[0079]** One should realize that the method described has the effect that if the one extends the bandwidth of the Wilkinson power divider by a large amount using the method described, then the maximum obtainable isolation decreases correspondingly. Thus, in general, an appropriate tradeoff has to be found.

**[0080]** For the general case, an N-way Wilkinson power divider, similar results as in the above derived case for N = 2 may be obtained. The results will be given below.

**[0081]** It should be noted, that in this general case, the theoratical value of $Z_w$ for the ordinary Wilkinson is $Z_w = \sqrt{N}\cdot Z_0$. This means that the necessary $Z_w$ for the improved Wilkinson is bounded by $Z_0/\sqrt{N} < Z_w < \infty$, i.e. $0 < k < 1/\sqrt{N}$ (since k

$= Z_0/Z_w$). In the following, it is assumed that $Z_w$ and k are within these bounds.

**[0082]** The input impedance becomes

$$Z_{11} = (1/N)\cdot Z_w(Z_0+jZ_w\tan\beta 1)/(Z_w+jZ_0\tan\beta 1)$$

so

$$\mathrm{Re}\{Z_{11}\}/Z_0 = (1/N)\cdot(1+\tan^2\theta)/(1+k^2\tan^2\theta)$$

and

$$\mathrm{Im}\{Z_{11}\}/Z_0 = (1/N)\cdot Z_w\tan\theta(1-k^2)/(1+k^2\tan^2\theta)$$

**[0083]** To find $2\Delta f$, set $\mathrm{Re}\{Z_{11}\}/Z_0 = 1$, which results in

$$\tan^2\theta = (N-1)/(1-Nk^2) \Rightarrow \tan\theta = \pm\sqrt{(N-1)}/\sqrt{(1-Nk^2)}$$

i.e.

$$\theta = \pm\arctan[\sqrt{(N-1)}/\sqrt{(1-Nk^2)}]$$

and so

$$2\Delta f = 2f_0(1-2\mathrm{Arctan}[\sqrt{(N-1)}/\sqrt{(1-Nk^2)}]/\pi)$$

**[0084]** As $k\to 0$, $2\Delta f$ becomes $2f_0(1-2\mathrm{Arctan}[\sqrt{(N-1)}]/\pi)$, so for increasing N, the maximum obtainable bandwidth improvement decreases.

**[0085]** To find $\alpha$, we calculate $\mathrm{Im}\{Z_{11}\}$ at $f = f_0 \pm \Delta f$, which results in

$$\begin{aligned}\mathrm{Im}\{Z_{11}\} &= (1/N)\cdot Z_w\tan\theta(1-k^2)/(1+k^2\tan^2\theta)\\ &= Z_w\tan\theta[(1-Nk^2)/N]\\ &= \pm Z_w\sqrt{[(N-1)(1-Nk^2)]}/N\\ &= \pm Z_0\sqrt{[(N-1)(1/k^2-N)]}/N\end{aligned}$$

so

$$\begin{aligned}\alpha &= \sqrt{[(N-1)(1/k^2-N]}/N\\ &= \sqrt{[(N-1)((Z_w/Z_0)^2-N)]}/N\end{aligned}$$

**[0086]** Observe that for N = 2, the above results reduce to the previously derived result for the 2-way Wilkinson. The same equations as for the 2-way Wilkinson is used to calculate L and C.

**[0087]** In summary, the steps for improving the bandwidth of Wilkinson power dividers using the method described can be summarized as follows:

a) choosing a desired $2\Delta f$;

(b) calculating the required k (i.e. $Z_0/Z_w$) and $\alpha$ from equation (20) and (23); and

(c) calculating L and C from equations (32) and (34) (using equation (33) instead of (34) if $\Delta f \ll f_0$)

**[0088]** Although the present invention has been described with respect to preferred embodiments thereof, it should be understood that many modifications can be performed without departing from the scope of the invention as defined by the appended claims.

**[0089]** Particularly, although a series connection of a single capacitor and inductor has been shown, a plurality of inductors and capacitors may be used instead.

**[0090]** Moreover, the invention is not restricted to an equal Wilkinson power divider circuit, but can be applied as well to an unequal Wilkinson power divider circuit.

## Claims

**1.** A Wilkinson power divider circuit, comprising:

a plurality of N transmission lines (TRL1, TRL2, ..., TRLN), N being an integer equal to or greater than 2, having a respective length of $1 = \lambda_0/4$ at a center frequency $f_0$ , where $\lambda_0$ is the wavelength at fo, and respective line impedances $Z_w$;

said plurality of N transmission lines (TRL1, TRL2, ..., TRLN) being connected to a first port (P1) at a respective first end, to a respective second port (P2, P3, ..., PN+1) at a respective second end, and via a respective resistor ($R_1$, ..., $R_N$) to a node (O) at the respective second end;

**characterized** by

an LC-circuit (50) comprising at least an inductor having an inductance L and a capacitor having a capacitance C connected in series between said first port (P1) and said first ends of said plurality of N transmission lines (TRL1, TRL2, ..., TRLN) having its resonance frequency $f_r$ at or near said center frequency $f_0$.

**2.** The Wilkinson power divider circuit according to claim 1, wherein said resistors ($R_1$, ..., $R_N$) have all the same impedance value $Z_w$.

**3.** The Wilkinson power divider circuit according to claim 1 or 2, wherein the resonance frequency $f_r$ is given by

$$f_r = \omega_r/2\pi = 1/\sqrt{((1-\varepsilon^2)LC)} = 2\pi f_0/\sqrt{(1-\varepsilon^2)}$$

$\varepsilon$ being $\Delta f/f_0$, $2\Delta f$ the desired isolation bandwith at minimum isolation, C being the capacitance of the capacitor, and L the inductance of the inductor.

**4.** The Wilkinson power divider circuit according to claim 3, wherein the impedance value $Z_w$ is given by

$$Z_w = Z_0/k$$

with k being obtainable from

$$2\Delta f = 2f_0(1-2\text{Arctan}[(\sqrt{(N-1)}/\sqrt{(1-Nk^2)})]/\pi).$$

**5.** The Wilkinson power divider circuit according to claim 4, wherein said inductance L and capacitance C are given by

$$L = \alpha Z_0/2\varepsilon\omega_0$$

$$C = 2\varepsilon/\omega_0\alpha Z_0 \text{ (for } \varepsilon \ll 1\text{) or } C = 2\varepsilon/(1-\varepsilon^2)\omega_0\alpha Z_0$$

with $\alpha = \sqrt{[(N-1)(1/k^2-N)]}/N$.

**6.** A design method for designing a Wilkinson power divider circuit according to at least one of the preceeding claims, comprising the steps of:

a) choosing a desired $2\Delta f$ equal to the required isolation bandwidth;

(b) calculating $k = Z_0/Z_w$ from $2\Delta f = 2f_0(1-2\mathrm{Arctan}[(\sqrt{(N-1)}/\sqrt{(1-Nk^2)})]/\pi)$; Zw from $Z_w = Z_0/k$; and calculating $\alpha$ from $\alpha = \sqrt{[(N-1)(1/k^2-N)]}/N$; and

(c) calculating the inductance L and the capacity C from $L = \alpha Z_0/2\varepsilon\omega_0$ and $C = 2\varepsilon/\omega_0\alpha Z_0$ (for $\varepsilon << 1$) or $C = 2\varepsilon/(1-\varepsilon^2)\omega_0\alpha Z_0$, respectively.

FIG 1

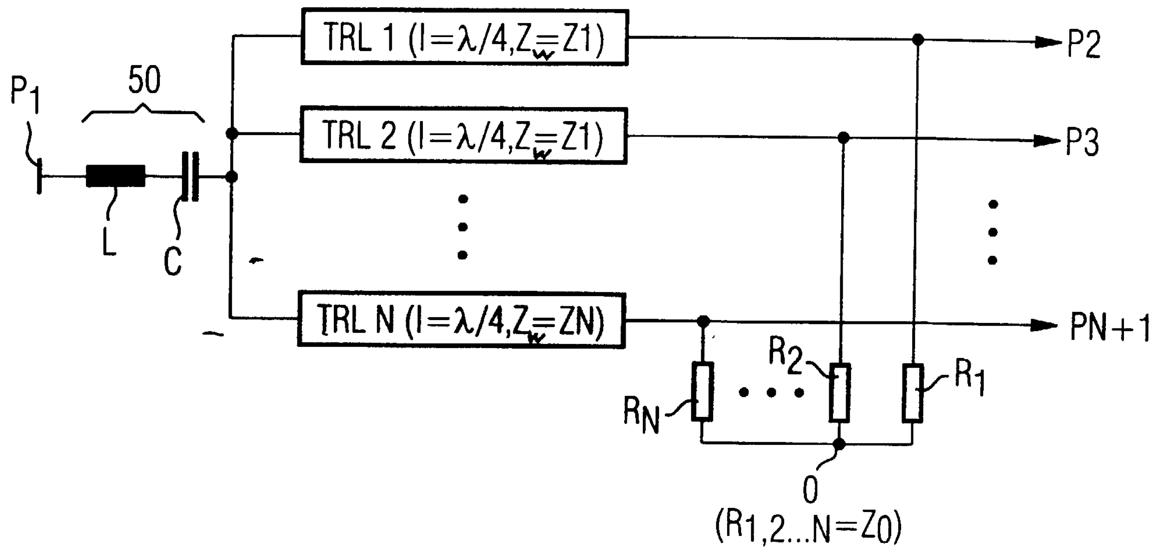

P1
50
L
C
TRL 1 (l=λ/4,Zw=Z1)
TRL 2 (l=λ/4,Zw=Z1)
TRL N (l=λ/4,Zw=ZN)
P2
P3
PN+1
RN
R2
R1
0
(R1,2...N=Z0)

FIG 2

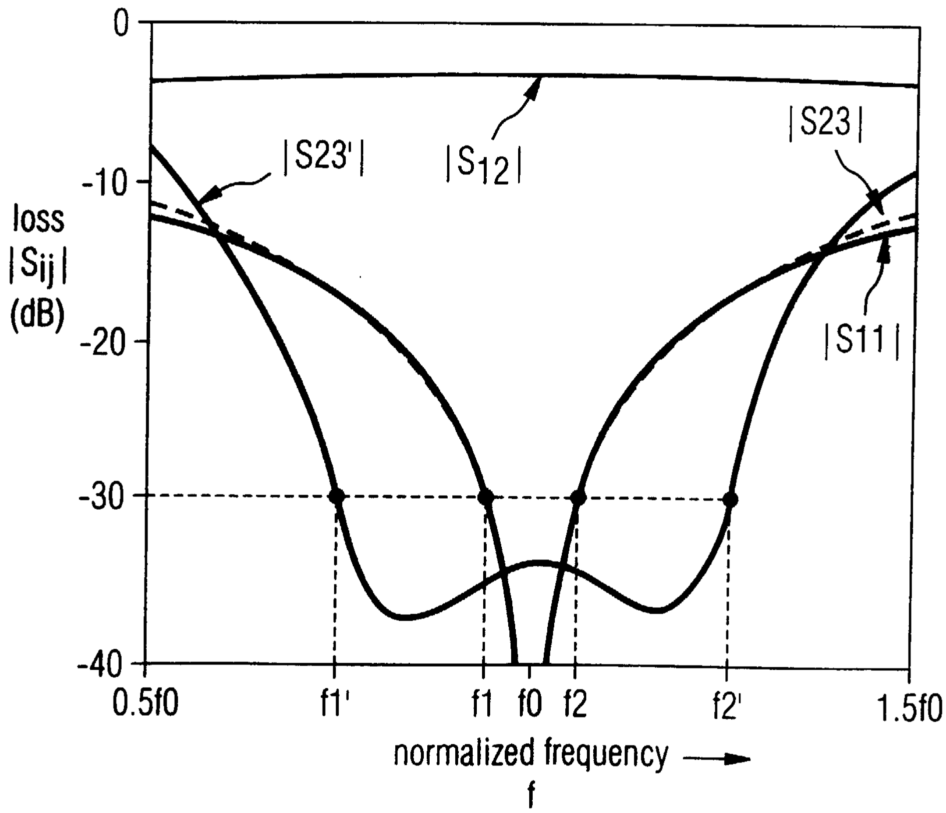

0
-10
-20
-30
-40
loss
|Sij|
(dB)
|S23'|
|S12|
|S23|
|S11|
0.5f0
f1'
f1
f0
f2
f2'
1.5f0
normalized frequency
f

FIG 3

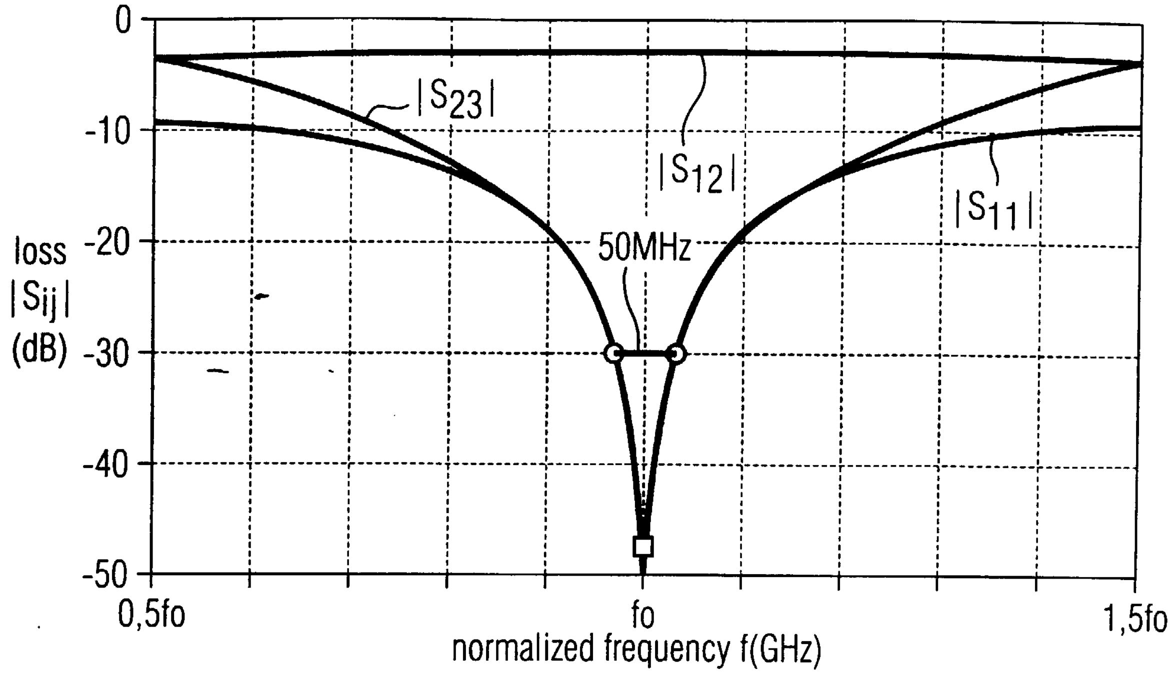

0
-10
-20
-30
-40
-50
loss
|Sij|
(dB)
|S23|
|S12|
|S11|
50MHz
0,5fo
fo
1,5fo
normalized frequency f(GHz)

FIG 4

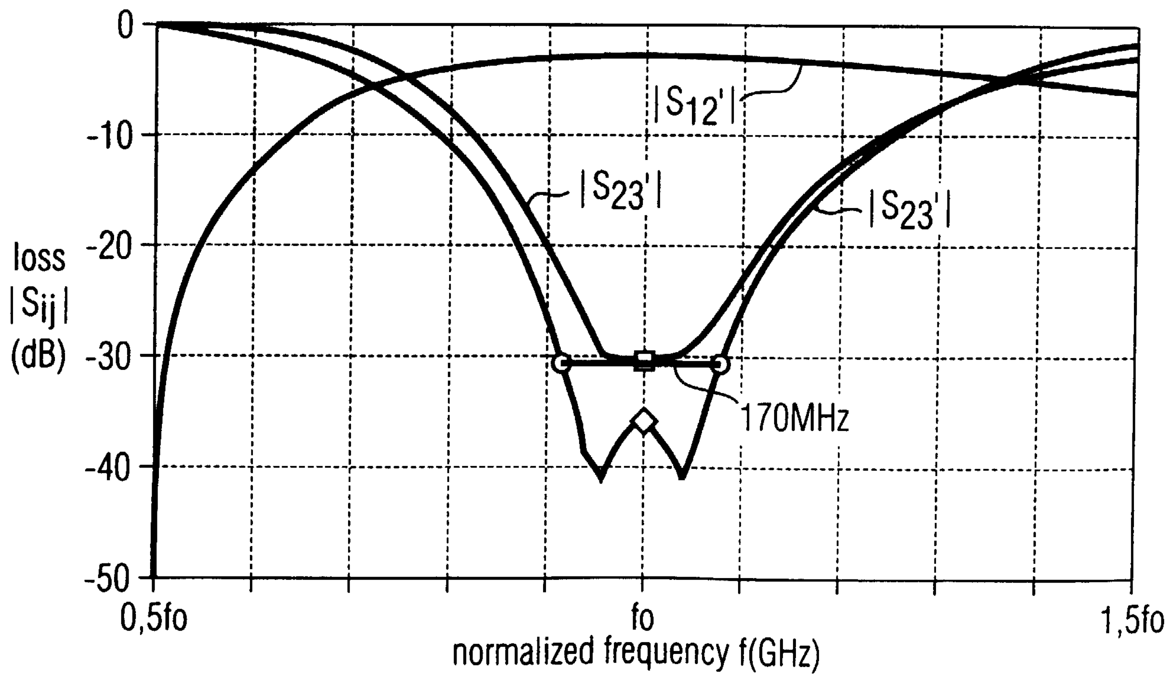
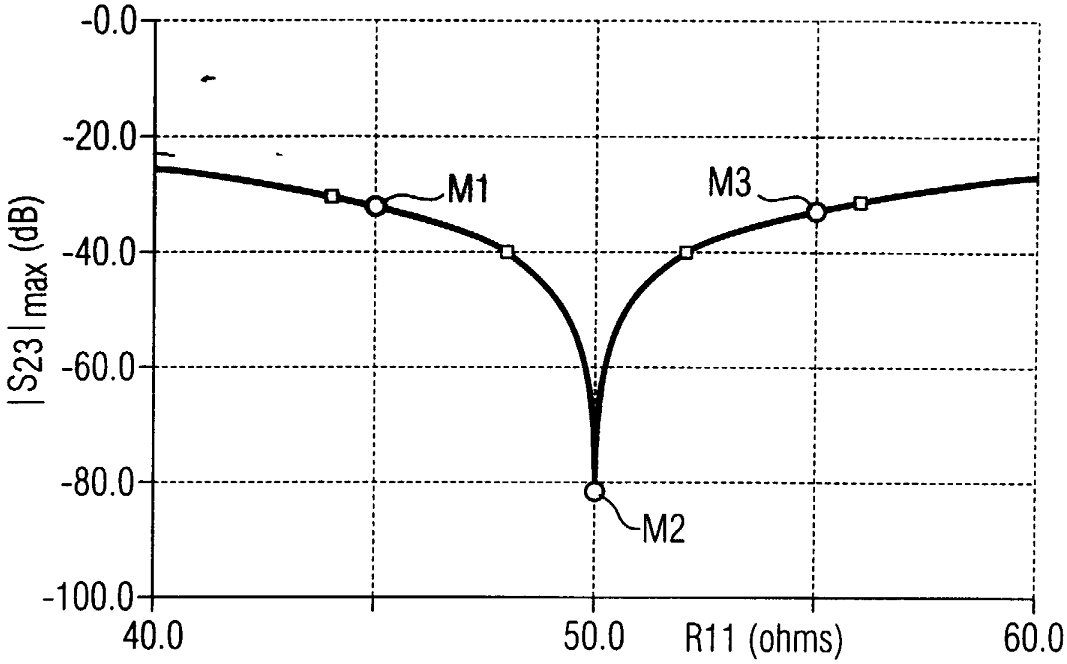
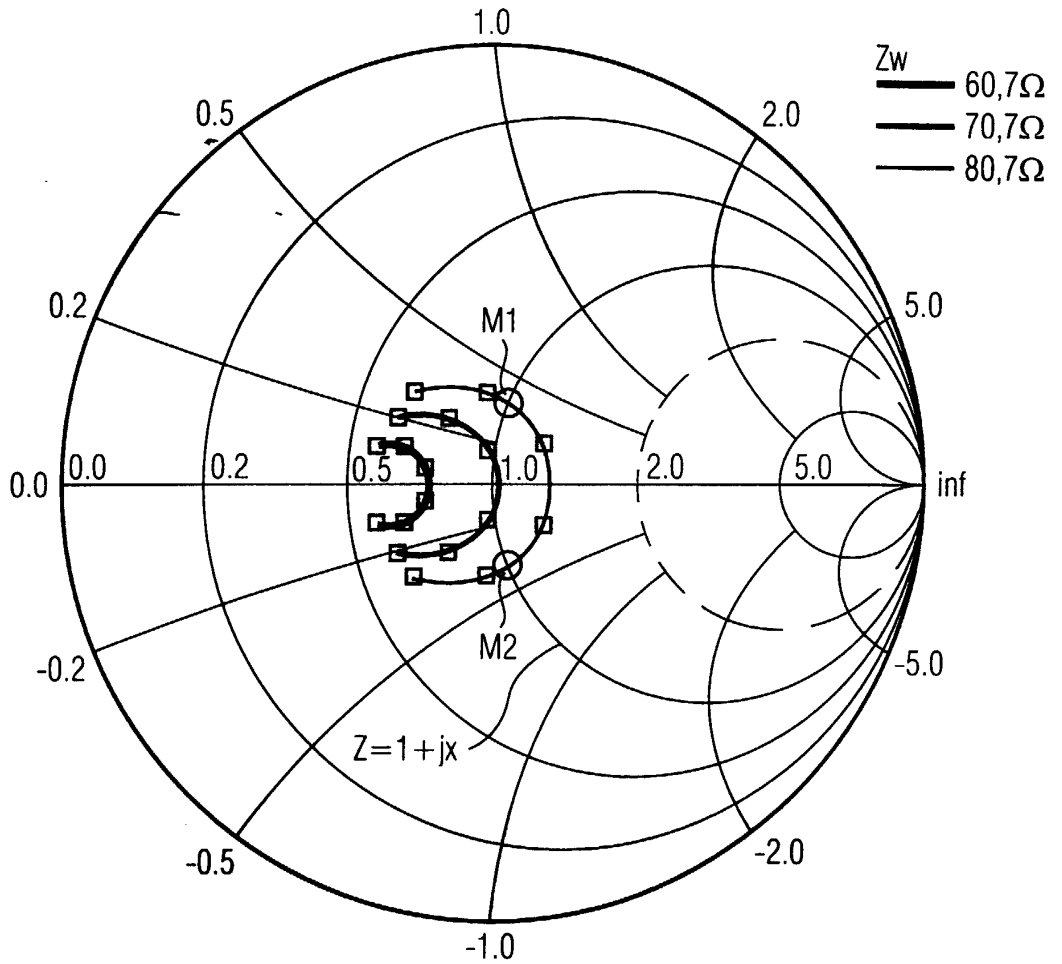

0
-10
-20
-30
-40
-50
loss
|Sij|
(dB)
|S12'|
|S23'|
|S23'|
170MHz
0,5fo
fo
1,5fo
normalized frequency f(GHz)

FIG 5




M1 R11= 45.0000000 value= -31.6205623
M2 R11= 50.0000000 value= -82.4397493
M3 R11= 55.0000000 value= -32.4375447

# FIG 6




Frequency 200.0 to 800.0 MHz

M1 Zw= 80.7000000 Frequency= 357.000000 r= 0.99983569 x= 0.38894913

M2 Zw= 80.7000000 Frequency= 643.000000 r= 0.99983569 x= 0.38894913

FIG 7

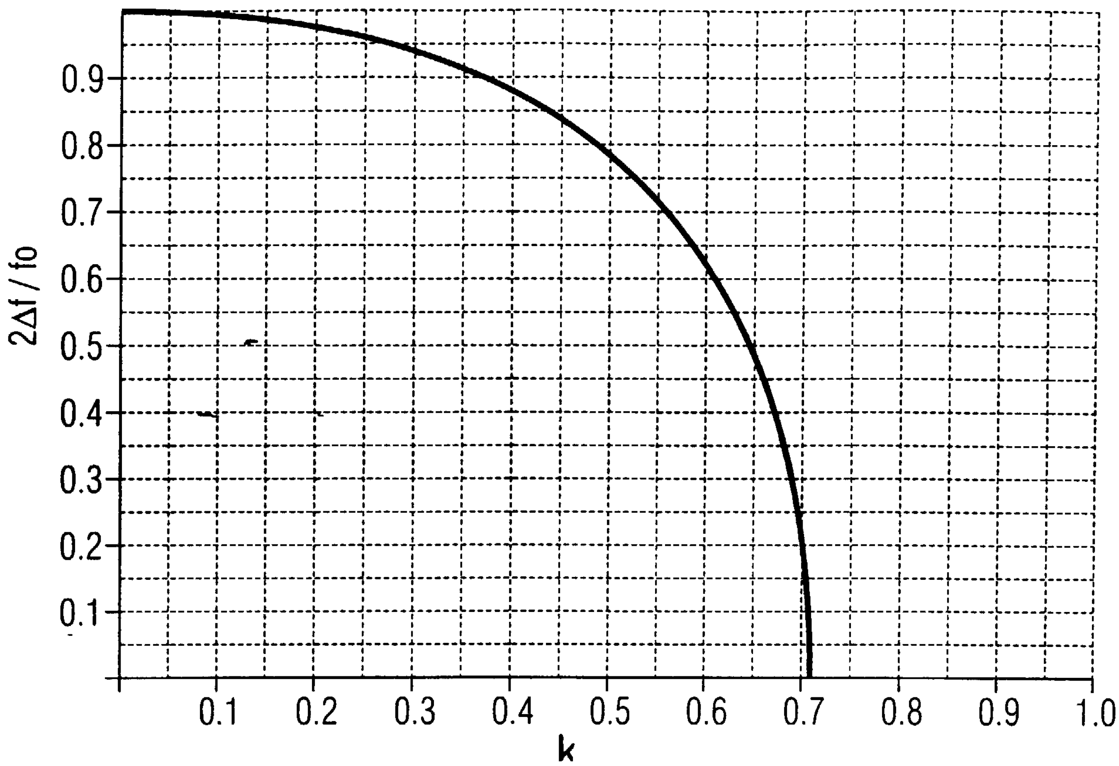

0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
2Δf / fo
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
k

FIG 8

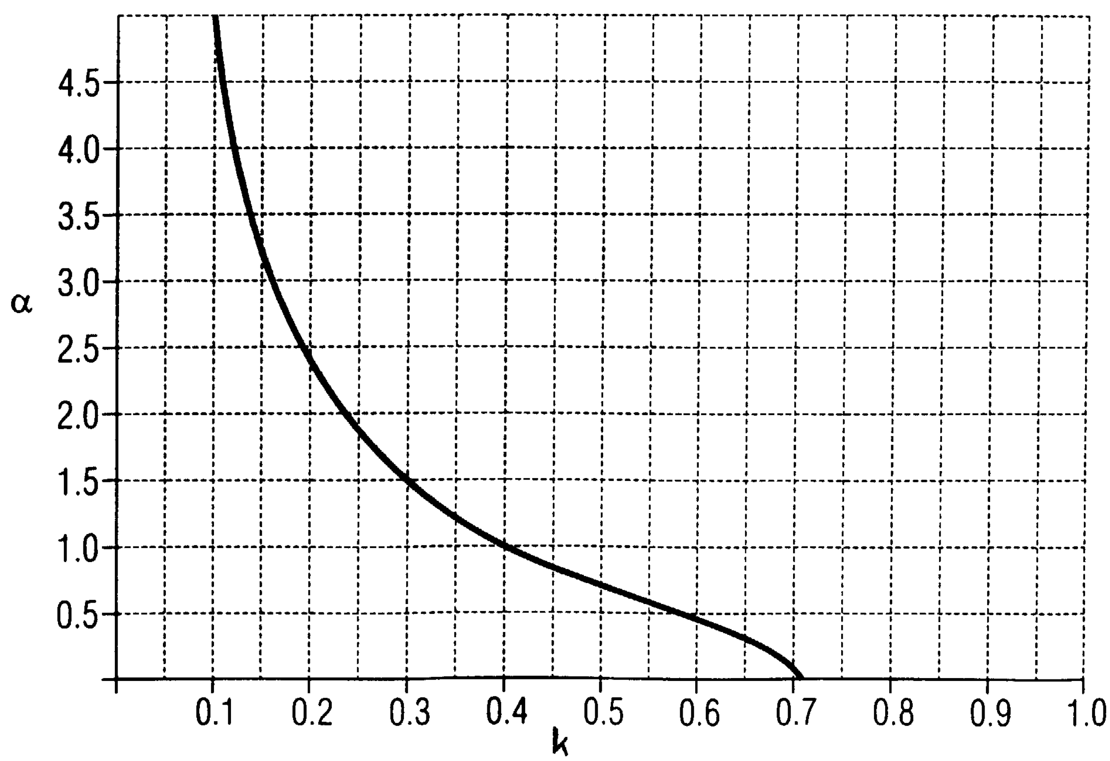
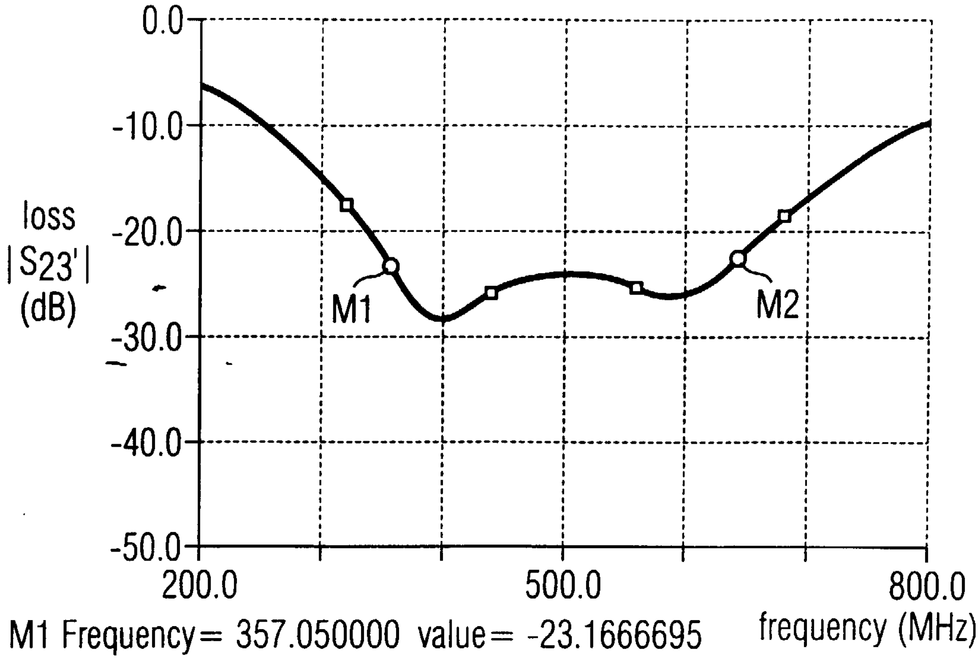
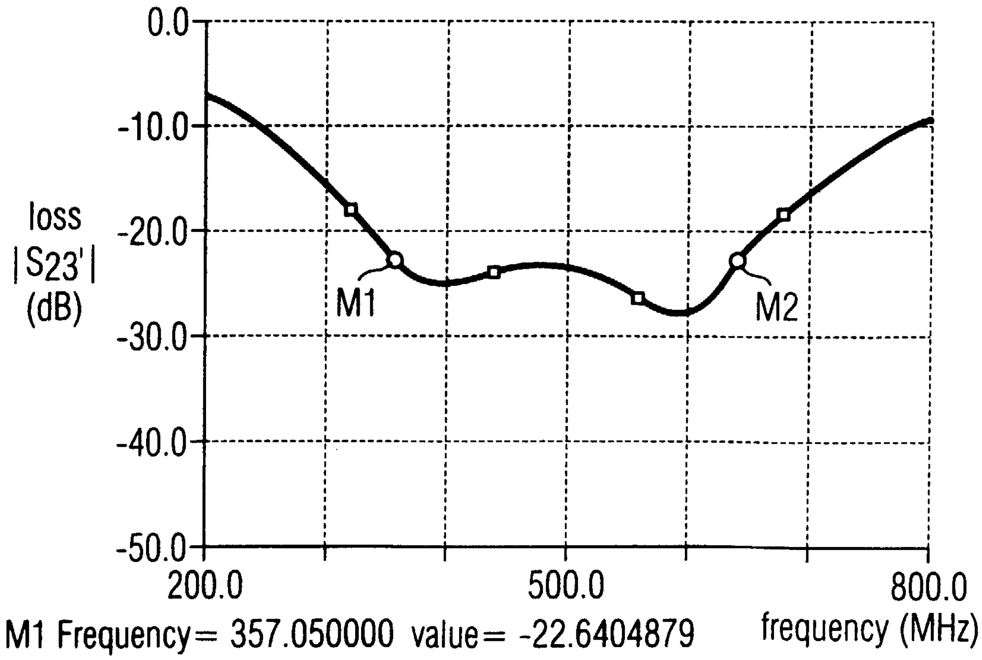

4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
α
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
k

FIG 9 L=10.82nH C=9.36pF



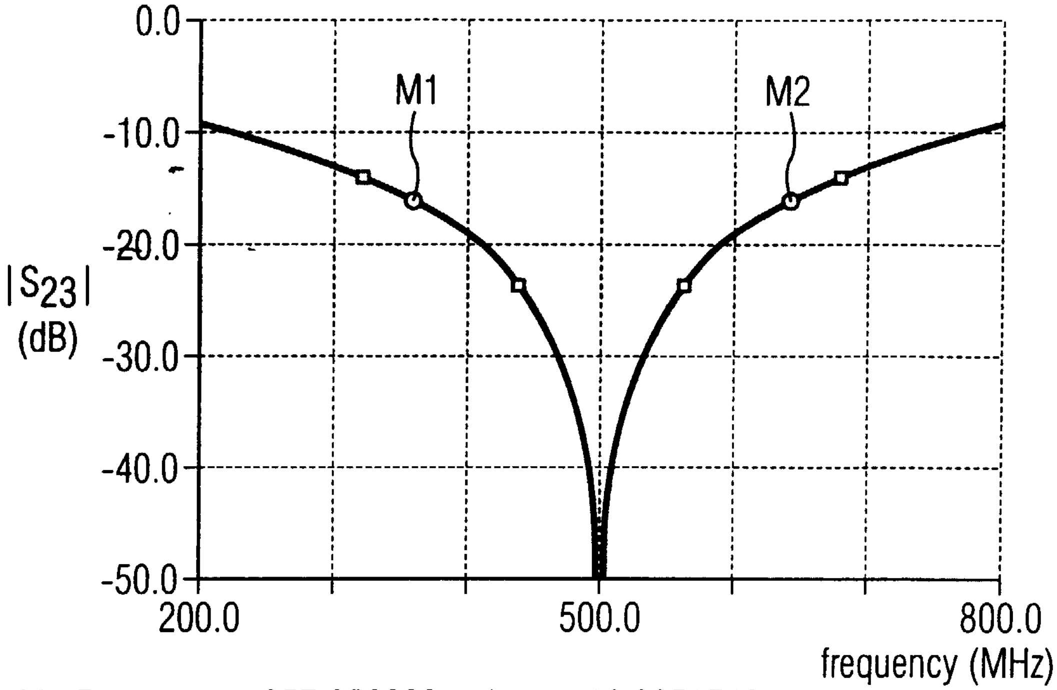

M1 Frequency= 357.050000 value= -23.1666695
M2 Frequency= 642.950000 value= -22.0586219

FIG 10 L=10.82nH C=10.19pF



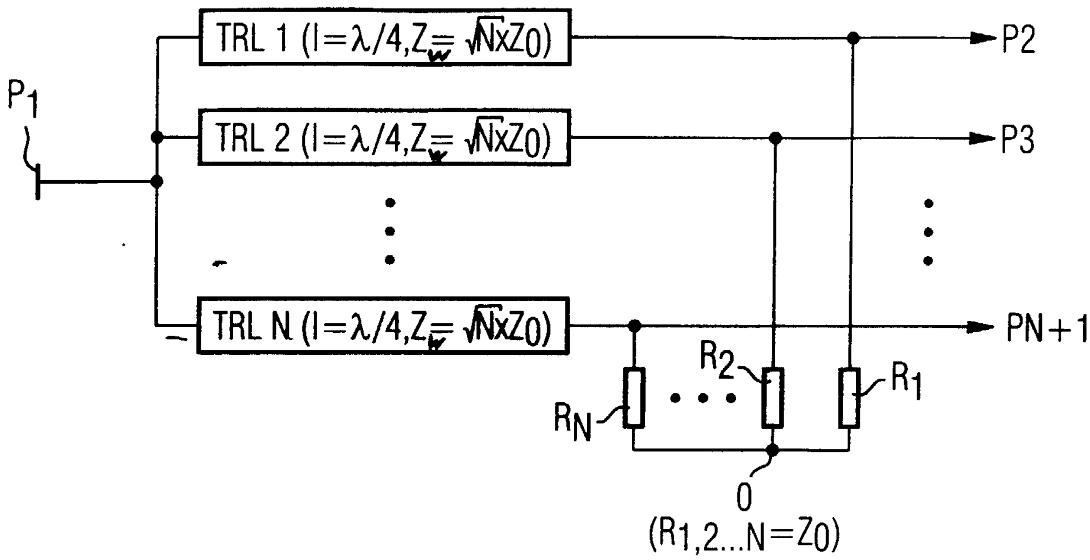

M1 Frequency= 357.050000 value= -22.6404879
M2 Frequency= 642.950000 value= -22.6286451

FIG 11

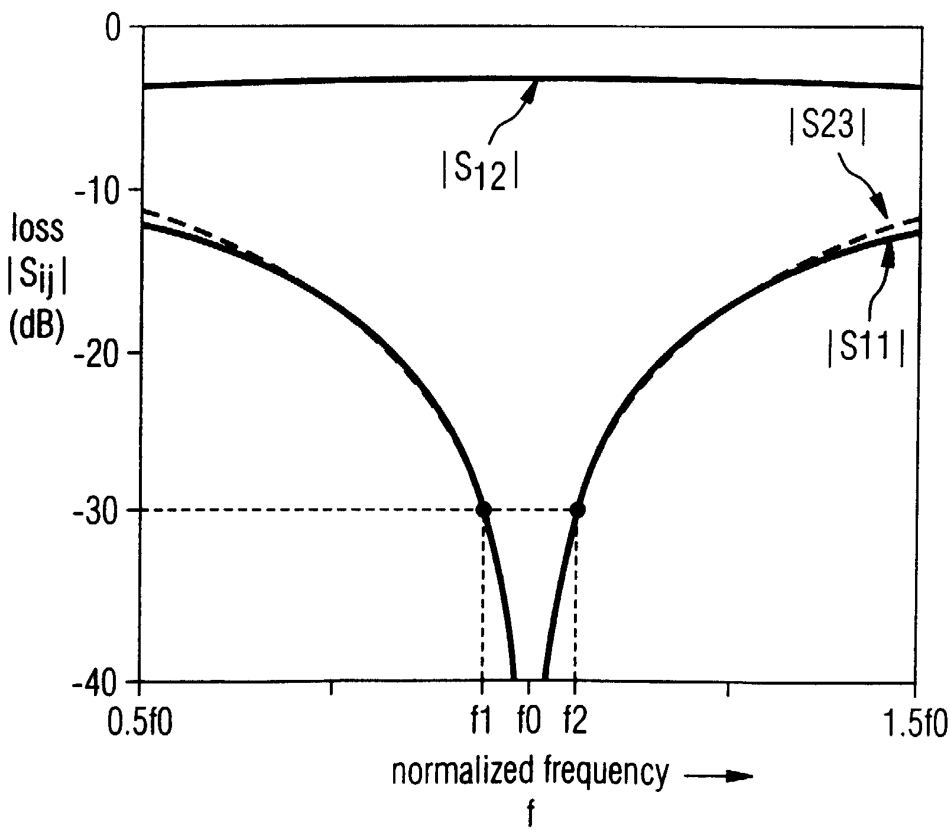



M1 Frequency= 357.050000 value= -16.0251746
M2 Frequency= 642.950000 value= -16.0251746

## FIG 12


P1
TRL 1 (l=λ/4,Zw= √NxZ0)
TRL 2 (l=λ/4,Zw= √NxZ0)
TRL N (l=λ/4,Zw= √NxZ0)
P2
P3
PN+1
R2
R1
RN
0
(R1,2...N=Z0)

## FIG 13


0
-10
-20
-30
-40
loss
|Sij|
(dB)
|S12|
|S23|
|S11|
0.5f0
f1
f0
f2
1.5f0
normalized frequency
f

European Patent Office

# EUROPEAN SEARCH REPORT

Application Number
EP 98 12 0556

## DOCUMENTS CONSIDERED TO BE RELEVANT

| Category | Citation of document with indication, where appropriate, of relevant passages | Relevant to claim | CLASSIFICATION OF THE APPLICATION (Int.Cl.6) |
|---|---|---|---|
| X<br><br><br><br>Y | PATENT ABSTRACTS OF JAPAN<br>vol. 5, no. 121 (E-068), 5 August 1981<br>-& JP 56 058310 A (TDK CORP), 21 May 1981<br>* abstract *<br>--- | 1<br><br><br><br>2 | H01P5/16 |
| Y | A.A.M. SALEH: "PLANAR ELECTRICALLY SYMMETRIC N-WAY HYBRID POWER DIVIDERS/COMBINERS"<br>IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES.,<br>vol. 28, no. 6, June 1980, pages 555-563, XP002095839<br>NEW YORK US<br>* page 555, right-hand column, line 20 - page 556, left-hand column, line 13; figures 1-5 *<br>--- | 2 | |
| A | GB 2 282 008 A (HUGHES AIRCRAFT COMPANY) 22 March 1995<br>* page 4, line 17 - page 6, line 8; figure 1 *<br>----- | 1 | |

| TECHNICAL FIELDS SEARCHED (Int.Cl.6) |
|---|
| H01P |

The present search report has been drawn up for all claims

| Place of search | Date of completion of the search | Examiner |
|---|---|---|
| THE HAGUE | 8 March 1999 | Den Otter, A |

CATEGORY OF CITED DOCUMENTS

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EPO FORM 1503 03.82 (P04C01)

## ANNEX TO THE EUROPEAN SEARCH REPORT ON EUROPEAN PATENT APPLICATION NO. EP 98 12 0556

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08-03-1999

| Patent document cited in search report | Publication date | Patent family member(s) | Publication date |
|---|---|---|---|
| GB 2282008 A | 22-03-1995 | US 5467063 A | 14-11-1995 |
| | | FR 2710471 A | 31-03-1995 |

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For more details about this annex : see Official Journal of the European Patent Office, No. 12/82