(19) Europäisches Patentamt
European Patent Office
Office européen des brevets

(11) EP 2 333 185 A1

(12) # EUROPEAN PATENT APPLICATION

(43) Date of publication:
15.06.2011 Bulletin 2011/24

(51) Int Cl.:
E04C 5/00 *(2006.01)* ***E01D 21/00*** *(2006.01)*

(21) Application number: **10015211.5**

(22) Date of filing: **01.12.2010**

(84) Designated Contracting States:
AL AT BE BG CH CY CZ DE DK EE ES FI FR GB GR HR HU IE IS IT LI LT LU LV MC MK MT NL NO PL PT RO RS SE SI SK SM TR
Designated Extension States:
BA ME

(30) Priority: **01.12.2009 EP 09014900**

(71) Applicant: **Prof. Dr.-Ing. Bulicek + Ingenieure**
94032 Passau (DE)

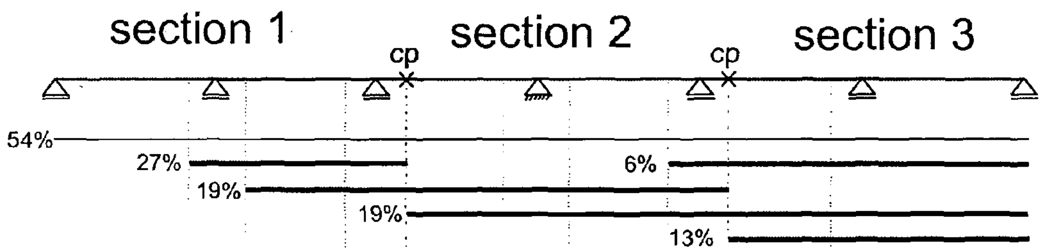
(72) Inventors:
- **Bulicek, Hans**
  94032 Passau (DE)
- **Seipelt, Siegfried**
  9410 Untergriesbach (DE)

(74) Representative: **Kitzhofer, Thomas**
Prinz & Partner
Rundfunkplatz 2
80335 München (DE)

(54) **Method of drafting and building a prestressed concrete structure**

(57) A method of drafting and building a prestressed concrete structure comprises the steps of a) modeling the problem of finding optimum parameters of prestressing strands in the prestressed concrete structure in a mathematical optimization problem; b) using a preferably gradient-based numerical search method to solve the mathematical optimization problem; and c) building the prestressed concrete structure in accordance with the results of step b). A prestressed concrete structure, in particular a bridge, is drafted and built according to this drafting and building method.

Fig. 6



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Description

[0001] The invention relates to a method of drafting and building a prestressed concrete structure, especially in the field of concrete bridge engineering. The invention further relates to a prestressed concrete structure.

[0002] Adequate drafts of prestressed concrete bridges have to fulfill all the prestressing requirements, which are to be met for a prestressed cross section as optimally as possible along the entire length of the structure. Due to the fact that the parameters of prestressing, such as the run and force of prestressing strands in a section, affect the stresses in all other sections, an improvement of the draft of prestressing of concrete bridges can, up until now, only be performed on the basis of iteration and the experienced data of the drafting engineer. If there is no success with reference to a constant quality of the draft over the entire length of the structure, there are excessive compressive stresses and, as the case may be, likewise adverse tensile stresses distributed over wide ranges of the structure as well as a higher demand for the total amount of prestressing steel.

[0003] Accordingly, there is a demand for an elaborate, iterative search for an optimal prestressing, especially within statically indeterminate load-bearing systems of sectionably-erected bridges. Therefore, it is an object of the invention to provide an improved method of drafting concrete elements with optimum prestressing, especially in the field of concrete bridge engineering, that can be performed by the use of computers.

[0004] According to the invention, the above object is met by a method of drafting a prestressed concrete structure comprising the features of claim 1. Advantageous and expedient embodiments of the invention are given in the dependent claims.

[0005] The method of drafting and building a prestressed concrete structure according to the invention comprises the steps of:

a) modeling the problem of finding optimum parameters of prestressing strands in the prestressed concrete structure in a mathematical optimization problem;

b) using a preferably gradient-based numerical search method to solve the mathematical optimization problem; and

c) building the prestressed concrete structure in accordance with the results of step b).

[0006] The method according to the invention links technical engineering knowledge with numerical search methods in a unique manner. By the use of the method according to the invention, the draft of the prestressing of concrete bridges can be carried out by the use of a computer; and thereby, the optimal distribution of prestressing forces and the optimal run of strands and tendons, respectively, within the relevant span-section can be found in order to avoid excessive compressive stresses and the occurrence of adverse tensile stresses under the considered load-level over the entire length of the structure.

[0007] Thus, the invention provides, for the first time, a computerizable procedure for the draft of the prestressing of concrete bridges based on non-linear numerical methods for optimization. Hereby, the prestressing force required for each section, as well as the run of the prestressing strands with reference to their length, elevation within the cross section and curvature, can be found and optimized.

[0008] The primary criterion of optimization is to achieve a minimum of the total amount of the prestressing steel necessary. Thereby, an improvement of the overall draft is achieved since excessive compressive stresses as well as adverse tensile stresses, as often seen in the case of a non-optimized draft of prestressing over wide ranges of the structure, are avoided.

[0009] Besides the economic advantages of an optimized prestressing, there are, above all, additional advantages concerning the draft of the construction. Those include more balanced and lower average concrete stresses compared to the results of a draft of the prestressing done "by hand". Therefore, a lower creep-induced loss of prestressing forces is to be considered; and, the probability of the appearance of cracks is reduced.

[0010] Another benefit is that due to the fact that the draft of prestressing strongly integrates the effects of the statically indeterminate prestressing-induced bending moments, there is a reduced susceptibility of the load-bearing structure against irregularities concerning dimension accuracy. This is due to the fact that the integral value of statically indeterminate bending moments is relatively stable with respect to possible local irregularities such as scattered discrepancies of installation dimensions with regard to the prestressing tendons.

[0011] It is expected that by means of the proposed method, harmonic, economic and statically useful prestressing parameters will be achieved for real applications, which fulfill all prestressing requirements in the best possible way along the entire length of the structures concerned.

[0012] According to the preferred embodiment of the invention, in above-mentioned step a) of the method a constrained non-linear optimization problem is modeled, including an objective function to be optimized, equality constraints and inequality constraints.

[0013] According to a particularly advantageous aspect of the invention, the mechanical equations used for the objective function and/or for the equality and/or inequality constraints are formulated based on principles of virtual work, instead of using finite element data. In fact, the complicated involvement of a finite element method has prevented civil engineers from using a mathematical optimization method for drafting prestressed concrete structures to date, especially in the field of concrete bridge engineering. By contrast, virtual work principles can be embedded into an optimization procedure much easier than a finite element method. The use of virtual work principles leads to a single (albeit very large) optimization equation capturing all relevant constraints. Since this equation is continuous enough and therefore differentiable at each point, any common gradient-based fast numerical search method can be utilized. This makes the method according to the invention both variable and fast.

[0014] The invention also provides a prestressed concrete structure, in particular a bridge. The bridge is drafted and built according to the method as defined in the invention.

[0015] Details of the invention will become apparent from the following description and from the accompanying drawings to which reference is made. In the drawings:

- Figure 1 shows a fictional construction-section 1 divided into sections 1 to i for the formulation of the mechanical equations;
- Figure 2 shows known prestressing-induced virtual moments and unknown elevation of strands and statically determinate prestressing-induced moments;
- Figure 3 shows a range for strands with variable and fixed anchoring points;
- Figure 4 shows a possible scheme of strands with variable and set points of anchorage fixture;
- Figure 5 shows the run of strands referring to the center of gravity of the superstructure's cross section;
- Figure 6 shows optimized lengths and forces of prestressing strands referring to Table 1; and
- Figure 7 shows statically indeterminate prestressing-induced bending moments (dimensionless).

I. PRINCIPLES OF OPTIMIZATION

I.1 GUIDING PRINCIPLE OF OPTIMIZATION

[0016] In the course of finding solutions within the scope of engineering and applied sciences, often minimal or maximal values of results are to be found with the least amount of effort in calculation. The search for an extreme value under consideration of the given boundary conditions and constraints represents the essential guiding principle of optimization.

I.2 PROCEDURE OF OPTIMIZATION

[0017] Since there are only few analytical solutions possible for optimization problems, the mathematical calculation is mostly based on numerical search methods. Such methods are distinguished into gradient-based methods on the one side, and direct methods on the other side (see Gekeler, E.W., "Mathematische Methoden zur Mechanik," Springer Verlag, December 2006, and Alt,W., "Nichtlineare Optimierung," Vieweg Verlag, May 2002).

[0018] In order to solve the problem at hand, a gradient-based method was chosen since the advantages of the direct methods could not be utilized and the underlying objective function, which is described in more detail in chapter II.3, is differentiable within the whole domain and therefore, the gradient detectable. Especially due to the high number of unknowns, the so-called "Interior Point Method" (see Vanderbei, R.J., "An interior point code for quadratic programming," Technical Report SQR 94-15, Princeton University, 1994, and Vanderbei, R.J., and Shanno, D., "An interior point algorithm for nonconvex nonlinear programming," Technical Report SQR 97-21, Princeton University, 1999) is referred to.

[0019] It should be noted that, in general, the mentioned procedures are only able to detect local extremes. However, by the use of a suitable starting point variation, the probability of finding a global extreme is increased and even guaranteed in individual cases.

[0020] The interior point algorithm solves a constrained optimization by combining constraints and the objective function f(x) through the use of a barrier-function, $q_k B(x)$ (see Bronstein, I.N., Semendjajew, K.A., Musiol, G., and Mühlig, H., "Taschenbuch der Mathematik," Harri Deutsch Verlag, June 2000). The function guarantees that the solution lies within the feasible area. The barrier-function B(x), with the additional parameter q_k, becomes larger the closer it is to the edge of an admissible area so that the barriers of this area cannot be overstepped.

[0021] Specifically, the general constrained optimization problem is first converted to the standard form of a non-linear optimization problem:

$$f(x) = min! \quad \text{and} \quad h(x) = 0,\ g(x) \geq 0 \qquad (1)$$

$$H_{q_k}(\boldsymbol{x}) = f(\boldsymbol{x}) + q_k \textstyle\sum_{k=1}^{m} B(x) \qquad (2)$$

[0022] The objective function f(x) represents the objective function which is to be optimized depending on the unknown vector x; h(**x**) represents the equality constraints and g(**x**) the inequality constraints.

[0023] The inequality constraints are replaced by a system of equalities in order to avoid an inequality system.

[0024] With the necessary Karush-Kuhn-Tucker condition (equation 3) we get a system of non-linear equations.

$$\nabla H_{q_k}(x) - y^T A\,(x) = 0 \quad \text{and} \quad h(x) = 0 \qquad (3)$$

$$B(x) = \ln\left(g_i(x)\right)\ \forall x \in\ M \qquad (4)$$

[0025] The vector y^T describes the langrangian multipliers; and, the matrix *A* (*x*) consists of the partial gradients of the equality constraints h(x).

[0026] The system can then be solved by standard algorithms for non-linear systems of equations, where the "Newton Method" or "Newton Algorithm" (see Bronstein, I.N., Semendjajew, K.A., Musiol, G., and Mühlig, H., "Taschenbuch der Mathematik," Harri Deutsch Verlag, June 2000, and Rade, L., and Westergren, B., "Mathematische Formeln," Springer Verlag, April 2000) is a proved, stable and often used solver. It is also possible to use standard gradient algorithms; however, there is one important drawback. Using the standard algorithms, it is important that the start-vector for the search of the optimum lies within the feasible domain.

II. APPLYING OPTIMIZATION TO THE DRAFT OF PRESTRESSING OF CONCRETE BRIDGES

II.1 PRINCIPLE STRATEGY AND GOALS OF OPTIMIZATION

[0027] In order to formulate the task of optimization, all mechanical equations needed for the objective function as well as for the equality and inequality constraints are to be formed and transformed into a system which is computationally manageable.

[0028] Those equations should be formulated directly based on the principles of virtual work. This is more variable and requires less time for calculation due to their small amount of data volume compared to the finite element method. Hereby, for example, the consideration of the effects due to cracked concrete would even be possible if the relevant moment-curvature-relation is known for a section.

$$W_{int} = \int_V \delta\sigma \cdot \epsilon\, dV \quad \rightarrow \quad \int_x \delta M \cdot \frac{M}{EI}\, dx \qquad (5)$$

$$W_{ext} = \Delta \cdot \delta F \qquad (6)$$

[0029] The virtual operational equations can be formulated for the bending girder proceeding from continuum according to W_{int} in equation 5. By comparing internal and external work-values, deformations on the basis of real and virtual bending moments can be calculated.

[0030] Statically indeterminate load-bearing systems are first to be transformed into statically determinate systems through the introduction of virtual hinges. Proceeding from the compatibility condition for continuous beams, after which the bending line has no kinks, the basic equations for the optimization of statically indeterminate systems are developed. The resulting virtual back-twisting bending moments correlate to the statically indeterminates.

[0031] The goal of the optimization is to find the optimal relevant prestressing parameters along the entire structure. In this case, the prestressing cables act effectively and therefore, the total amount of the necessary prestressing steel will be minimized. During the search for an optimal prestressing, the prestressing force, run, as well as length of the cables or strands, respectively, have to be considered as variable according to the relevant conditions of statics.

[0032] Furthermore, the following particular conditions should be taken into account: sectionable erection of the superstructure (where applicable); time-dependent loss of prestressing forces due to creep and shrinkage of concrete; primary constraints regulated by national codes, such as the limitation of longitudinal, compressive and tensile stresses under defined load-levels; friction-induced loss of prestressing force along the cables; and, the consideration of uncoupled, continuous cables along wide ranges of the structure. In part, such as in the sectionable erection of superstructures, this can already be taken into account within the formulation of the principle mechanical equations.

[0033] Further conditions, such as, for example, the limitation of the longitudinal stresses due to bending moments and axial forces, have to be formulated as inequality constraints limiting the possible range of solutions.

II.2 FORMULATION OF THE OPTIMIZATION EQUATIONS

[0034] The general strategy is shown in more detail in the following by way of an example of a fictional 3-span statically indeterminate bridge superstructure which is considered as to be erected in two sections due to economic reasons.

[0035] In the case of prestressed concrete bridges, all primary parameters of prestressing depend on the load-induced bending moments and axial forces as well as on prestressing-induced bending-moments themselves. The basic equation can therefore be derived from the course of bending moments and axial forces resulting from load and prestressing effects taking into account all additional effects which are to be considered.

[0036] The statically determinate and the statically indeterminate parts of prestressing, as well as the time-dependent variation of bending moments due to the different age of the concrete within the sequential sections of erection, should be included in -that basic equation.

[0037] For the numerical description, the superstructure has to be divided into small subsequent sections. By way of example, **Figure 1** shows a fictional construction-section 1 divided into sections 1 to i for the formulation of the mechanical equations.

[0038] After that, the mechanical properties of the cross section are assigned for the relevant sections of the superstructure. The force of the relevant prestressing strands and the strand's elevation within the cross section correlate to that system as well.

[0039] The statically determinate moment-parts, $x[i] \cdot a_{jk}[i] \cdot \sigma_0[i]$, as well as the statically indeterminate moment-parts, Mj[i], are represented by a vector of length k, where j is the order of statically indeterminacy. k corresponds to the number of sections introduced. **Figure 2** shows the known prestressing-induced virtual moments (left) and the unknown elevation of strands and the statically determinate prestressing-induced moments, respectively (right)

[0040] The statically determinate (real) parts of the prestressing moments cause twistings Δ_{10} and Δ_{20} within the integrated virtual hinges. Δ_{10} here corresponds to the real twisting at the virtual hinges due to the prestressing. Due to compatible conditions, those must be compensated by the statically indeterminate twisting parts resulting from the virtual bending moments Δ_{11}, Δ_{22}, Δ_{12} and Δ_{21}. The value Δ_{11} hereby corresponds to the back-twisting at the first interior column due to the virtual bending moments M1 with the normalized value 1; and, Δ_{12} corresponds to the back-twisting at the first interior column due to the virtual bending moment M2.

[0041] The integrals in equation 5 are calculated in two steps with respect to a more effective numerical handling. First, the related statically determinate moments proceeding from the prestressing are calculated, resulting in the form of a trapezoid-shaped area (see $\frac{M}{EI}$ (00) in **Figure 2** on the right). $\frac{M}{EI}$ (11) coresponds to the effects of the statically-indeterminate bending moment M1 at the first interior column. $\frac{M}{EI}$ (00), as well as the virtual related parts of bending moments $\frac{\mathbf{M}}{\mathbf{EI}}$ (11) and $\frac{\mathbf{M}}{\mathbf{EI}}$ (22) are described by a vector with a reduced length of k-1 since, instead of the sections, here only small area-parts are used. Incidentally, this is the first part of the integration procedure. It is mentioned that the vectors for the statically indeterminate parts of the prestressing already include all unknowns such as the length x (i) of the strands as well as the amount a_{jj}[i] or the force, respectively, of the appropriate prestressing strand.

[0042] The vectorial description leads to a very simple formulation of a global equation for the prestressing-induced bending moments under the consideration of all conditions. The indices always refer to the part of the superstructure being considered or active.

$$\frac{\mathrm{M}}{\mathrm{EI}}(00)[i,i+1] = \left(\frac{x[i]\cdot a_{jk}[i]\cdot \sigma_0[i]}{EI[i]} + \frac{x[i+1]\cdot a_{jk}[i+1]\cdot \sigma_0[i+1]}{EI[i+1]}\right)\cdot\frac{\Delta[i,i+1]}{2} \quad (6)$$

$$\frac{\mathrm{M}}{\mathrm{EI}}(11)[\mathrm{i},\mathrm{i}+1] = \left(\frac{\mathrm{M1[i]}}{\mathrm{EI[i]}} + \frac{\mathrm{M1[i+1]}}{\mathrm{EI[i+1]}}\right)\cdot\frac{\Delta[\mathrm{i,i+1}]}{2} \quad (7)$$

[0043] For the case in which there is not a sectionable erection of the superstructure, the statically indeterminate parts of the prestressing over the interior columns can already be calculated depending on the unknown elevation x[i] of the strand within the cross section and the unknown amount of prestressing force a_{jk}[i] by the use of a simple matrix scheme. The multiplication of vectors here also corresponds to the formulation of integrals necessary within the principle of virtual works.

[0044] Through the internal product of the vectors, the second step of the numerical integration procedure of the integrals in (5) is carried out. It should, however, be mentioned that its scaling is performed by 1/EI; this is, however, simplified in the course of the matrix multiplication. As a result, there is an equation for the determination of prestressing-caused, statically-indeterminate bending moments over the interior columns of the load-bearing system.

$$(\mathrm{X1_{end}},\mathrm{X2_{end}})^{\mathrm{T}} = \begin{pmatrix} -\overrightarrow{\mathrm{M/EI(00)}}\cdot\overrightarrow{\mathrm{M/EI(11)}} \\ -\overrightarrow{\mathrm{M/EI(00)}}\cdot\overrightarrow{\mathrm{M/EI(22)}} \end{pmatrix}\cdot\begin{pmatrix} \overrightarrow{\mathrm{M/EI(11)}}\cdot\overrightarrow{\mathrm{M/EI(11)}} & \overrightarrow{\mathrm{M/EI(11)}}\cdot\overrightarrow{\mathrm{M/EI(22)}} \\ \overrightarrow{\mathrm{M/EI(22)}}\cdot\overrightarrow{\mathrm{M/EI(11)}} & \overrightarrow{\mathrm{M/EI(22)}}\cdot\overrightarrow{\mathrm{M/EI(11)}} \end{pmatrix}^{-1} \quad (8)$$

$$\frac{\Delta_{10}}{\mathrm{EI}} = \frac{\mathrm{M}}{\mathrm{EI}}(00)\cdot\frac{\mathrm{M}}{\mathrm{EI}}(11) \text{ and } \frac{\Delta_{11}}{\mathrm{EI}} = \frac{\mathrm{M}}{\mathrm{EI}}(11)\cdot\frac{\mathrm{M}}{\mathrm{EI}}(11) \quad (9)$$

[0045] $X1_{end}$ coresponds to the actual value of the bending moment, M1, and represents nothing more than the statically indeterminate part of the prestressing-induced bending moments over the first interior column and for $X2_{end}$ correspondingly.

[0046] It should be mentioned once more that the calculated, statically indeterminate parts X1 and X2 are now functions depending on the unknowns such as elevation x[i] and force ajj[i] of the relevant-strand-section. The multiplication of the values, X1 and X2, respectively, with the normalized course of the bending moments, M1 and M2, respectively, results in the run of the statically indeterminate parts for each considered hinge.

[0047] In the case of a sectionable erection, it also has to be considered that only a certain part of the prestressing elements is active within the relevant section. Only this part is responsible for the effect of statically indeterminate prestressing-induced bending moments within this erected section. The time-dependent variation of bending moments due to creep and shrinkage of the concrete can be determined, for example, by the use of the variation-factor C_{cr}=(1-c_v) from Trost (see Mehlhorn, G., "Der Ingenieurbau - Bemessung", Ernst & Sohn, April 1999).

[0048] In the following section, all prestressing elements which do not fall within the relevant section are to be considered as inactive.

[0049] By way of example, **Figure 3** shows a range for strands with variable and fixed anchoring points. In section 1 of erection only the strands a11, a21, a31, a2, and a3 are active. All others are inactive. In the first section, a statically indeterminate part, $x1_{sec1}$, is only possible over the first interior column. Since it deals with a product of vectors, the following equation provides values for the prestressing-induced bending moments depending on the unknowns.

$$X1_{sec1}(x[i], a_{jj}[i]) = \frac{-\overrightarrow{\frac{M}{EI}(00)} \cdot \overrightarrow{\frac{M}{EI}(11)}}{\overrightarrow{\frac{M}{EI}(11)} \cdot \overrightarrow{\frac{M}{EI}(11)}} \qquad (10)$$

[0050] Within the construction-section 2, the additional parts over the first interior column as well as the parts over the second interior column are to be calculated. From that, a solution vector with the result values $(X1_{sec2}, X2_{sec2})^T$ can be derived.

[0051] The final statically indeterminate parts, which are to be multiplied with the triangle-functions in **Figure 2,** accrue from the superposition of the section-values with the values based on a static system without any section joints and without section-part $X1_{end}$ taking into account the above-mentioned variation-factor due to the time-dependent variation of bending moments.

[0052] The following equation provides for the statically indeterminate prestressing-induced bending moments over the first interior column in the considered 3-span-system.

$$X1 = (X1_{sec1} + X1_{sec2}) \cdot (1 - C_{cr}) + X1_{end} \cdot C_{cr} \qquad (11)$$

[0053] Through a superposition of the statically determinate and statically indeterminate parts of prestressing, a global equation can be formulated for the total prestressing-induced bending moments depending on the unknown length, run and force of the strands.

[0054] The effects resulting from the erection procedure are fully included in that equation. The latter is a basis for the formulation of the objective function as well as the constraints.

[0055] Taking into account the statically indeterminate parts of the prestressing, the position of the centers of moments, calculated on the basis of the dead-load of the bridge, can vary marginally. Therefore, it is recommended to define the possible anchorage-points of the strands surrounding the above-mentioned corrected centers of moments. Hereby, the optimal effect of prestressing is made possible. However, the numerical effort is comparably high.

II.3 FORMULATION OF THE OBJECTIVE FUNCTION AND CONSTRAINTS

[0056] The objective function as well as the constraints can now easily be formulated as vector-equations for each relevant section. It is for example $\Sigma\, a_{jj}[i] = \min$.

[0057] For instance, conditions for edge stresses, which also proceed from bending equations, can be formulated as constraints. Likewise, relations between subsequent strands, admissible curvatures of strands or non-negative constraints can also serve as constraints.

[0058] With the constraint after which the force of all serial strands to the right must always be larger or equal than the relevant previous strand to the left, the right side for the allocation of the prestressing anchorage fixture is set. This holds vice versa as well.

[0059] The solving of the problem can be conducted by the use of the above-mentioned algorithm. In order to assure a fast convergence of the solution-procedure, a start-vector for the prestressing which refers, for example, to the curvature of bending moments resulting from the dead-loads of a superstructure can be defined. The statically indeterminate parts of the prestressing can generally be neglected in the primary approach.

III. APPLICATION BY THE USE OF AN EXAMPLE AND INTERPRETATION OF THE RESULTS

III.1 DESCRIPTION OF THE EXAMPLE AND OF THE PRESTRESSING CONCEPT

[0060] The applicability of the shown method will be demonstrated qualitatively by the way of a prestressed sectionably-erected 6-span-highway-viaduct which is planned to be built in Germany (see Bulicek, H., "Comparison of the Application of Internal Prestressing Unbonded Tendons with Bonded Tendons by the example of the Schallermühle Viaduct, Innsbrucker Bautage 2009," V. 3, iup (Innsbruck University Press), Innsbruck (Austria), January 2009, pp. 157-168). The considered superstructure of the bridge is prestressed in the longitudinal direction by the use of internal, post-tensioned grouted tendons. The 240 m long structure with a multi-web T-beam cross-section will be erected in three sections, whereby span 3 and span 5 will house the relevant section joint and couple-points for the cables respectively. The span is approximately 32 m in the first and last span and 44 m in the four middle spans. At least 50 % of the amount of prestressing steel in the particular section should be run through that section (see thin line in **Figure 4** which shows a possible scheme of strands with variable and set points of anchorage fixture).

[0061] In general, the following decision has to be made before starting the optimization process. As a rule, there is a pre-defined choice of the length of the individual strands with respect to the construction process such as the possibility to set anchorage fixtures and couplings. In this case, only the run of the relevant strands and their force is variable with reference to the optimization. Alternatively, it is also possible to consider the length of the strands as well as the run of the strands as being variable (see thick lines in **Figure 4).** In the following, the latter is considered.

[0062] As is seen in **Figure 3,** the given section joints limit the possibility of varying the length of the strands. In all other areas, the allocation of the anchorages is variable with respect to the optimization process.

[0063] As already mentioned before with respect to the allocation of the prestressing anchorage fixture, as a constraint the force of all serial strands to the right must always be larger or equal than the relevant previous strand to the left. This holds vice versa as well.

[0064] Concerning the iteration, the solution which requires a minimal amount of prestressing steel is memorized for each section considered. This value is then used as a start value for the next section. The optimal length of the strands is strongly influenced by the statically indeterminate, prestressing-caused bending moments next to the actual run of the strands.

III.2 RESULTS OF THE CALCULATION

[0065] For the above-mentioned superstructure, the prestressing has been drafted by the use of this non-linear optimization procedure.

[0066] **Figures 5 and 6** show the relevant results qualitatively. The run of the prestressing strands is displayed for each section considered. The position of the center of gravity of the strands is also allocated automatically based on a minimum total amount of prestressing steel. However, in order to avoid prestressing strands, which are impracticably short, a minimum length should be defined within the definition of the constraints.

[0067] The following Table 1 indicates the relative amount of prestressing force in the relevant strands along the superstructure.

Tendon-no.		Dimensionless amount of prestressing force
A11, A12, A13		54 %
A2		0 %
	A21	27 %
A3		46 %
	A31	46 %
A4		38 %
	A41	38 %
A5		38 %
	A51	44 %
A6		38 %
	A61	38 %
A7		38 %

[0068] Now remaining is the important task of the drafting engineer to choose the relevant prestressing cables in order to cover the necessary amount of optimized prestressing forces along the entire superstructure and fit them into the cross section according to the optimized run, meanwhile taking into account the specific constraints of all the givens.

III.3 INTERPRETATION OF RESULTS

[0069] As it is seen in the numerical results, the statically indeterminate prestressing-induced bending moments (see **Figure 6** which relates to the values of Table 1) play an important role in the finding of an optimal conduction of the prestressing strands.

[0070] It also shows that for an optimized run of the prestressing strands, the latter must not always be conducted close to the edge of the cross section, depending on, of course, the givens of the relevant case. The finding of an optimal run of the prestressing strands "by hand", is therefore, especially for large bridges with variable cross sections, a very time-consuming endeavor requiring much instinctive feeling on the part of the drafting engineer with regard to the prestressing of concrete bridges.

[0071] **Figure 7** shows the statically indeterminate prestressing-induced bending moments (dimensionless).

IV. SUMMARY AND OUTLOOK

[0072] In its structure, the presented method for drafting and building a prestressed concrete structure, in particular a bridge, combines, on the one hand, engineering, constructional and mechanical knowledge with numerical search methods, on the other hand. In doing so, the performance is strongly influenced by several points which are to be considered from the constructional point of view. Above all, the scheme of selecting the prestressing strand sections (see Figure 4) is to be mentioned. This results in a reduction of the total number of unknowns due to the fact that the amount of prestressing steel elements is combined to prestressing strands at the individual intersections.

[0073] Generally, the structure of the presented method for drafting and building a prestressed concrete may be subdivided into 5 segments:

1. Setting up a mostly non-linear system of equations combining the effect of the prestressing with the longitudinal forces and the bending moments in the bridge structure. Not only can the principle of virtual forces (as described before) be utilized for this subtask as a numerical variant.

[0074] Setting up the system of equations may also be carried out using finite elements as basic elements. In doing so, merely the effect of the prestressing has to be numerically described through the curvature of the prestressing strands. Thus, the level of the strands is introduced into the equation in an indirect way.

[0075] The system of equations may also be formulated for the respective prestressing strands with the principle of the virtual forces but using spline functions (so-called "B-splines"). In doing so, each prestressing strand A_i is described as a B-spline. The form of the strand can be defined in most cases by means of few (but at least seven) test points. All method-related numerical differentiations and integrations are carried out then by the B-Spline functions. With this, the description of the prestressing strand position is possible with a lower number of supporting points whereby the numerical effort for solving the problem will crucially decrease.

2. Selecting a suitable, problem-related objective function. The solution behavior is greatly impacted by the selection of the objective function. In case of using function parts with oscillating functions courses, it may happen that a global solution cannot be found. In the context of the presented method, the sum of the prestressing strand areas $\Sigma_j \Sigma_i A_{ij}$ is used. This selection is only possible due to the innovative prestressing strand scheme. This is why the solution search is carried out on the basis low-order n-dimensional polynomials and the convergence is certain.

3. Formulating suitable constraints and boundary conditions which are applicable to the respective construction progress as well as the related construction method of the bridge structure. The constraints and boundary conditions allow considering, inter alia, the side of force introduction, the admissible bending radii of the tendons, friction-related losses along the respective prestressing strand as well as the normative constraints in the method. The side of force introduction may be defined by the condition $A_{21}(\cdot i) \geq A_{20}(i) \wedge A_{31}(i) \geq A_{30}$, for example. Again, the selected prestressing strand scheme allows using this type of boundary condition and permits a combination of the construction method with the system of equations.

4. Numerical solution of the system of equations with boundary conditions.
Here, classical gradient-based as well as stochastic search methods may be used. The structure of the problem, however, prefers the methods mentioned at first.

5. Evaluation of the results, and building the bridge based on the results.

Claims

1. A method of drafting and building a prestressed concrete structure, comprising the steps of:

   a) modeling the problem of finding optimum parameters of prestressing strands in the prestressed concrete structure in a mathematical optimization problem; and
   b) using a preferably gradient-based numerical search method to solve the mathematical optimization problem; and
   c) building the prestressed concrete structure in accordance with the results of step b).

2. The method according to claim 1, **characterized in that** in step a) a constrained non-linear optimization problem is modeled, including an objective function to be optimized, equality constraints and inequality constraints; the objective function preferably relating to the total amount of the necessary strands material, in particular steel, in the prestressed concrete structure.

3. The method according to claim 2, **characterized in that** mechanical equations used for the objective function and/or for the equality and/or inequality constraints are formulated based on principles of virtual work, preferably involving transforming statically indeterminate load-bearing systems into statically determinate systems through the introduction of virtual hinges.

4. The method according to claim 2, **characterized in that** mechanical equations used for the objective function and/or for the equality and/or inequality constraints are formulated based on using finite elements as basic elements, or based on principles of virtual forces and using spline functions.

5. The method according to any of claims 2 to 4, **characterized in that** the equality constraints and/or inequality constraints relate to relevant conditions of statics involving at least one of the following strand parameters: prestressing force, run, length; wherein the equality constraints and/or inequality constraints preferably further relate to at least one of the following particular conditions: sectionable erection of the structure; time-dependent loss of prestressing forces due to creep and shrinkage of concrete; limitation of longitudinal, compressive and tensile stresses under defined load-levels; friction-induced loss of prestressing force along the strands; consideration of uncoupled, continuous strands along wide ranges of the structure; limitation of the longitudinal stresses due to bending moments and axial forces.

6. The method according to any of the preceding claims, **characterized in that** step a) involves virtually dividing the structure into small subsequent sections for numerical description and assigning mechanical properties of the cross section for relevant sections of the structure, including the force of relevant prestressing strands and the strands' elevation within the cross section.

7. The method according to claim 2 and any of claims 3 to 6, **characterized in that** formulating a basic optimization equation in the optimization problem is derived from a course of bending moments and axial forces resulting from load and prestressing effects, wherein preferably at least one of the following is included in the basic equation: statically determinate and statically indeterminate parts of prestressing; time-dependent variation of bending moments due to different age of concrete within sequential sections of erection.

8. The method according to claim 7, **characterized in that** through a superposition of statically determinate and statically indeterminate parts of prestressing, a global equation is formulated for the total prestressing-induced bending moments depending on the unknown parameters of the strands, the global equation being the basis for formulating the objective function to be optimized.

9. The method according to claim 2 and any of claims 3 to 8, **characterized in that** one of the constraints is the condition that the force of all serial strands to the right must always be larger or equal than the relevant previous strand to the left, and vice versa, in order to set the right side or the left side, respectively, for an allocation of a prestressing anchorage fixture.

10. The method according to claim 2 and any of claims 3 to 9, **characterized in that** a scheme of strands, which is used to build the prestressed concrete structure, is devised according to the solved constrained non-linear optimization problem.

11. The method according to claim 2 and any of claims 3 to 10, **characterized in that** the inequality constraints are replaced by a system of equalities.

12. The method according to claim 2 and any of claims 3 to 11, **characterized in that** a system of non-linear equations is formed from the constrained non-linear optimization problem by using a necessary Karush-Kuhn-Tucker condition, the system of non-linear equations preferably being solved by using the Newton Method.

13. The method according to claim 12, **characterized in that** for solving the system of non-linear equations a start-vector for the prestressing which refers to the curvature of bending moments resulting from dead-loads of the structure is defined, and statically indeterminate parts of the prestressing are neglected in a primary approach.

14. A prestressed concrete structure, in particular a bridge, drafted and built according to a method as defined in one of claims 1 to 13.

Fig. 1

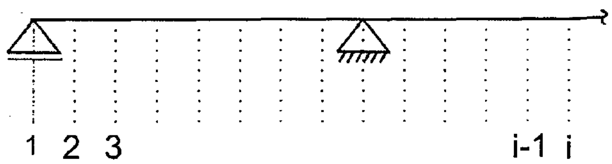


Fig. 3

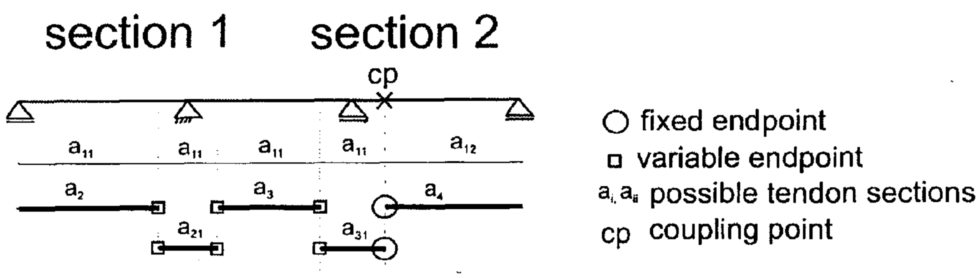


Fig. 4

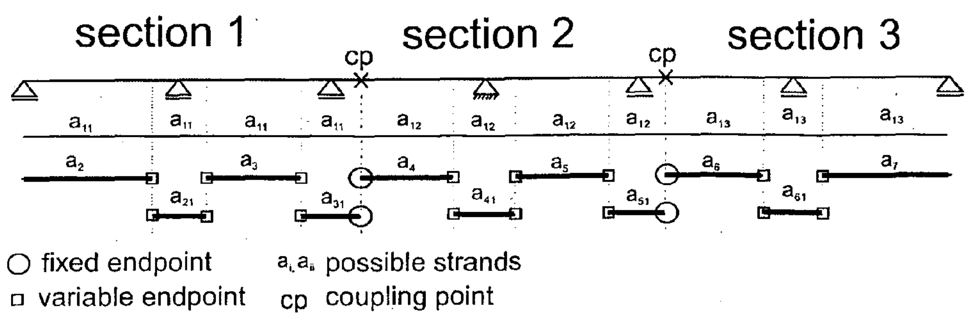


Fig. 2

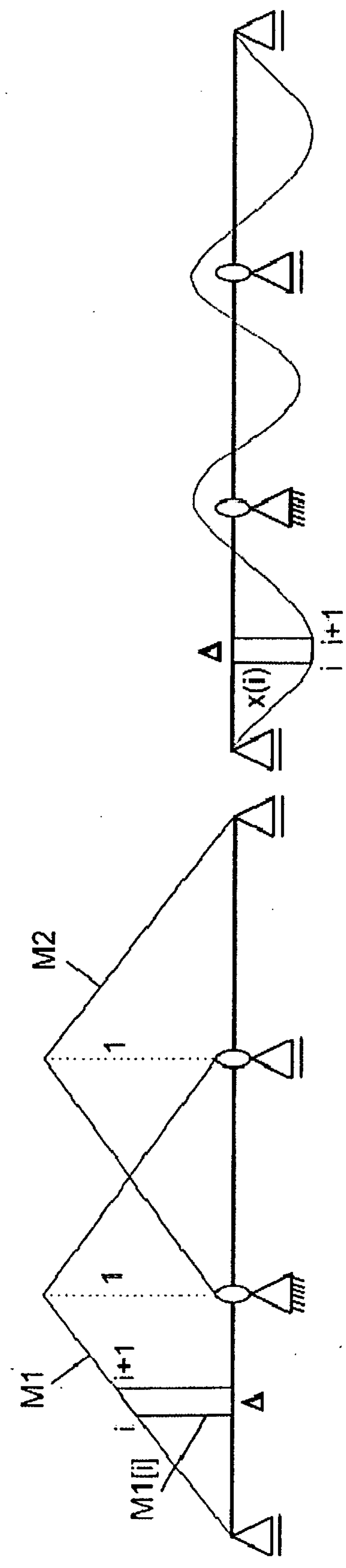


Fig. 5

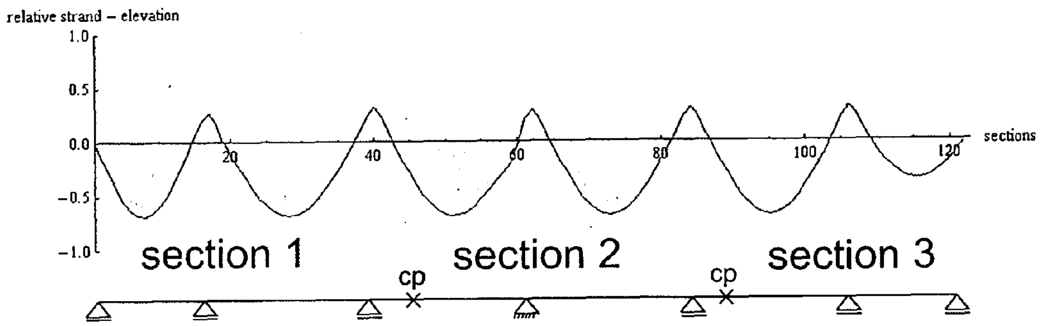


Fig. 6

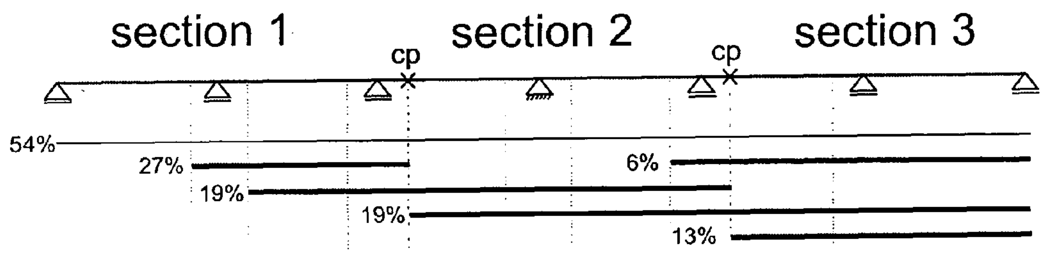
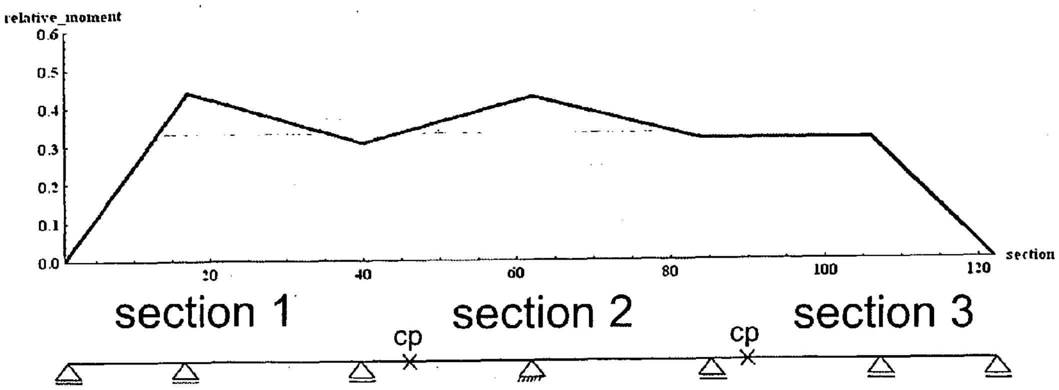


Fig. 7



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EUROPEAN SEARCH REPORT

Application Number
EP 10 01 5211

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X	APARICIO ET AL: "Computer aided Design of prestressed Concrete highway Bridges", COMPUTERS & STRUCTURES, vol. 60, no. 6, S0045, 31 December 1996 (1996-12-31), pages 957-969, XP002606802, Great Britain * page 961 - page 964; figure 10 *	1,14	INV. E04C5/00 E01D21/00
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TECHNICAL FIELDS SEARCHED (IPC)
E04C
E01D

The present search report has been drawn up for all claims

Place of search	Date of completion of the search	Examiner
Munich	15 March 2011	Saretta, Guido

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